

Mathematics (Subjective)**(GROUP-I)**

Time: 2:30 hours

SECTION-I

2. Write short answers of any eight parts from the following: (8x2=16)

- Define a complex number. Is 0 a complex number?
- Whether the set $\{0, -1\}$ is closed or not w.r.t addition and multiplication.
- Factorize: $3x^2 + 3y^2$
- Find multiplicative inverse of $-3 - 5i$
- Construct truth table of $\sim(p \rightarrow q) \rightarrow p$
- Define monoid.
- Find the matrix X if: $X \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$
- If A and B are square matrices of the same order, then explain why in general $(A + B)^2 \neq A^2 + 2AB + B^2$
- If $A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$, find $A(\bar{A})^t$
- Find four fourth roots of 81
- Use the remainder theorem to find the remainder when $x^3 - 2x^2 + 3x + 3$ is divided by $x - 3$
- If α, β are the roots of $3x^2 - 2x + 4 = 0$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

3. Write short answers of any eight parts from the following: (8x2=16)

- Define conditional equation.
- Resolve $\frac{x^2+15}{(x^4+2x+5)(x-1)}$ into partial fraction without finding constants.
- Find the first four terms of the sequence $a_n = \frac{n}{2n+1}$
- Determine whether -19 is a term of $17, 13, 9, \dots$
- Find the 5th term of the G.P $3, 6, 12, \dots$
- Sum the series $\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{5}{\sqrt{2}} + \dots + a_{13}$
- Prove from the first principle that ${}^n P_r = n \cdot {}^{n-1} P_{r-1}$
- Find the value of n when ${}^n C_{12} = {}^n C_6$
- Determine the probability of getting dots less than 5 when a die is rolled.
- Prove that $n! > 2^n - 1$ for $n = 4, 5$
- Calculate $(2.02)^4$ by means of binomial theorem.
- Expand $(1 + 2x)^{-1}$ up to 4 terms.

4. Write short answers of any nine parts from the following: (9x2=18)

- Write values of trigonometric functions for $\theta = \frac{-9}{2}\pi$.
- Prove that $t^2\theta - \cos^2\theta = \cot^2\theta \cos^2\theta$.

- iii. Prove that $\sin(\theta + 270^\circ) = -\cos\theta$.
- iv. Prove that $\sin 2\theta = 2\sin\theta \cos\theta$.
- v. Express $\sin 12^\circ \sin 46^\circ$ as sum or difference.
- vi. Write domain and range of $\cos x$.
- vii. Find period of $\sin \frac{x}{3}$.
- viii. Draw the graph of $\tan x$ for $x \in (0, \pi)$.
- ix. Prove that $r = (s - b)\tan \frac{\beta}{2}$.
- x. Write any two half angle formulae.
- xi. When angle between ground and sun is 30° , flag pole casts a shadow of $40m$ long. Find height of top of flag.
- xii. Show that $\cos(\sin^{-1}x) = \sqrt{1 - x^2}$.
- xiii. Solve the equation $4 \cos^2 x - 3 = 0$.

SECTION-II

Note: Attempt any three questions. Each question carries equal marks:

(10x3=30)

5.(a) If α and β are the roots of $x^2 - 3x + 5 = 0$, form the equation whose roots are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

(b) Find the rank of matrix $\begin{bmatrix} 1 & -1 & 2 & 1 \\ 2 & -6 & 5 & 1 \\ 3 & 5 & 4 & -3 \end{bmatrix}$

6. (a) Resolve $\frac{1}{(x-1)^2(x^2+2)}$ into partial fractions.

(b) Find six arithmetic means between 2 and 5.

7. (a) A die is thrown. Find the probability that the no. of dots on the top are prime numbers or odd numbers.

(b) If x is so small that its cube or higher powers can be neglected, show that $\sqrt{\frac{1-x}{1+x}} \approx 1 - x + \frac{1}{2}x^2$

8. (a) Solve the triangle ABC, given that $\alpha = 35^\circ 17'$, $\beta = 45^\circ 13'$, $b = 421$.

(b) Reduce $\cos^4 \theta$ to an expression involving only function of multiples of θ , raised to the first power.

9. (a) A circular wire of radius 6 cm is cut straightened and then bent so as to lie along the circumference of a hoop of radius 24 cm . Find the measure of the angle which it subtends at the center of the hoop.

(b) Prove that: $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} = \tan^{-1} \frac{9}{19}$