

Mathematics (Subjective)

(GROUP-II)

SECTION-I

(8x2=16)

2. Write short answers of any eight parts from the following:

- Does the set $\{1, -1\}$ possess closure property w. r. t multiplication? Construct the multiplication table.
- If $\frac{a}{b} = \frac{c}{d}$, prove that $ad = bc$
- Factorize $a^2 + 4b^2$
- Simplify by expressing in the form $a + bi$: $(2 + \sqrt{-3})(3 + \sqrt{-3})$
- If $B = \{1, 2, 3\}$ then write down the power set of B
- Determine whether the statement $p \rightarrow (q \rightarrow p)$ is a tautology or not.
- Under what conditions, the determinant of a square matrix A is zero. Write any two conditions.
- If $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, find the values of a and b .
- Determine whether the matrix $A = \begin{bmatrix} 1 & 1+i \\ 1-i & 2 \end{bmatrix}$ is hermitian matrix or skew-hermitian matrix.
- Solve the equation: $x^{-2} - 10 = 3x^{-1}$
- Find four fourth roots of 16.
- Show that the roots of equation will be rational $px^2 - (p-q)x - q = 0$

(8x2=16)

3. Write short answers of any eight parts from the following:

- Define an identity with example.
- Resolve into partial fraction $\frac{1}{x^2-1}$
- The 7th and 10th terms of an H.P are $\frac{1}{3}$ and $\frac{5}{21}$ respectively, find its 14th term.
- Find the sum of first 15 terms of geometric sequence $1, \frac{1}{3}, \frac{1}{9}, \dots$
- Insert two G.M's between 2 and 16.
- How many terms of the series $-7 + (-5) + (-3) + \dots$ amount to 65
- A card is drawn from a deck of 52 playing cards. What is the probability that it is a diamond card or an ace?
- Find n , if ${}^nC_8 = {}^nC_{12}$
- How many different 4-digit numbers can be formed out of the digits 1, 2, 3, 4, 5, 6, when no digit is repeated?
- Use mathematical induction to prove that $3 + 3.5 + 3.5^2 + \dots + 3.5^n = \frac{3(5^{n+1}-1)}{4}$ for $n = 1, 2$
- Calculate by means of binomial theorem $(2.02)^4$
- Expand upto 4 - terms $(1-x)^{1/2}$

(9x2=18)

4. Write short answers of any nine parts from the following:

- Find r , when $l = 56\text{cm}$, $\theta = 45^\circ$
- Verify that $\sin 2\theta = 2\sin\theta\cos\theta$ for $\theta = 45^\circ$
- Write the fundamental law of trigonometry.

- iv. Show that $\cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta$.
- v. Express $\sin 5x + \sin 7x$ as a product.
- vi. Define the period of trigonometric function.
- vii. Write down the domain and range of tangent function.
- viii. Find the period of $\sin \frac{x}{3}$.
- ix. Solve the right triangle ABC , in which $\gamma = 90^\circ$, $a = 3.28$, $b = 5.74$.
- x. Define half angle formulas for tangent.
- xi. Define Hero's formula.
- xii. Find the value of $\sin(\tan^{-1}(-1))$.
- xiii. Solve the equation $\sin 2x = \cos x$ where $x \in [0, 2\pi]$.

SECTION-II

(10x3=30)

Note: Attempt any three questions. Each question carries equal marks:

5.(a) Show that
$$\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$$

(b) Prove that $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$ will have equal roots if $c^2 = a^2m^2 + b^2$; $a \neq 0, b \neq 0$

6. (a) Resolve into partial fractions $\frac{6x^3+5x^2-7}{2x^2-x-1}$

(b) The A.M between the two numbers is 5 and their positive G.M. is 4 find the numbers.

7. (a) Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

(b) Find the coefficient of x^5 in the expansion of $\left(x^2 - \frac{3}{2x}\right)^{10}$

8. (a) Reduce $\sin^4 \theta$ to an expression involving only functions of multiples of θ raised to the first power.

(b) With usual notations, prove that $r = s \cdot \tan^{\alpha/2} \cdot \tan^{\beta/2} \cdot \tan^{\gamma/2}$

9. (a) If $\cot \theta = \frac{5}{2}$, and θ is in quadrant I, find the value of $\frac{3\sin \theta + 4\cos \theta}{\cos \theta - \sin \theta}$

(b) Prove that $\cos^{-1} \frac{63}{65} + 2\tan^{-1} \frac{1}{5} = \sin^{-1} \frac{3}{5}$