Roll No.________ *to be filled in by the candidate.* **(For all sessions)**

Mathematics (Essay Type)

Time: 2:30 Hours **Marks: 80**

Section -I

2. Write short answers of any eight parts from the following. **2x8=16**

i. Find the modulus of complex number $3+4i$.

ii. Simplify by justifying each step $\dfrac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}$ by writing properties.

iii. Factorize the expression $9a^2+16b^2$.

iv. Define absurdity and give one example.

v. Solve the system of linear equations. $\begin{aligned}4x_1+3x_2&=5\\3x_1-x_2&=7\end{aligned}$

vi. Find the value of x if $\begin{vmatrix}1 & 2 & 1\\2 & x & 2\\3 & 6 & x\end{vmatrix}=0$.

vii. Define Row Rank of a matrix.

viii. Solve the equation $x^{-2}-10=3x^{-1}$.

ix. If $A=\{1,2,3,4\}, B=\{3,4,5,6,7,8\}, C=\{5,6,7,9,10\}$ verify distributivity of union over intersection.

x. Find the inverse of the relation $\{(1,3),(2,5),(3,7),(4,9),(5,11)\}$.

xi. Use remainder theorem to find the remainder when x^3-x^2+5x+4 is divided by $x-2$.

xii. Find the roots of the equation $16x^2+8x+1=0$ by using quadratic formula.

3. Write short answers of any eight parts from the following. **2x8=16**

i. Resolve $\dfrac{1}{x^2-1}$ into partial fraction.

ii. Find 5^{th} term of Geometric progression G.P 2,6,12,.......

iii. Define Circular permutation.

iv. Expand $(4-3x)^{\frac{1}{2}}$ upto three terms.

v. If $\dfrac{1}{a},\dfrac{1}{b},\dfrac{1}{c}$ are in Arithmatic progression (A.P) show that common difference is $\dfrac{a-c}{2ac}$.

vi. If 5,6 are two Arithmatic Means (A.M) between "a" and "b". Find "a" and "b".

vii. If the numbers $\dfrac{1}{k},\dfrac{1}{2k+1},\dfrac{1}{4k-1}$ are in (H.P) Hormonic Progression, Find "K".

viii. How many words can be formed from the letters of PLAN" using all letters when no letter is to be repeated?

ix. If ${}^{n}c_{5}={}^{n}c_{4}$, where c stands for combination then find value of n.

x. Verify the inequality $n>2^n-1$ for integral values of $n=4,5$.

xi. If x is so small that its square and higher power cab be neglected, show that $\dfrac{1-x}{\sqrt{1-x}}=1-\dfrac{3}{2}x$.

xii. Prove that Hormonic Mean (H.M) between two numbers "a" and "b" is $\dfrac{2ab}{a+b}$.

4. Write short answers of any nine parts from the following. **2x9=18**

i. Prove the fundamental identity $\cos^2\theta+\sin^2\theta=1$.

ii. Verify the result $\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}$ for $\theta=30^0$.

iii. Show that $\frac{\cos 11^0 + \sin 11^0}{\cos 11^0 - \sin 11^0} = \tan 56^0$.

iv. Prove that $\cos 330^0 \sin 600^0 + \cos 120^0 \sin 150^0 = -1$.

v. Find the period of $\operatorname{cosec}(10x)$.

vi. Show that $\gamma = 4R \sin\frac{\alpha}{2} \sin\frac{\beta}{2} \sin\frac{\gamma}{2}$ with usual notation.

vii. Find the value of $\cos\left(\sin^{-1}\frac{1}{2}\right)$.

viii. Show that $\frac{\cot^2\theta - 1}{1 + \cot^2\theta} = 2\cos^2\theta - 1$.

ix. Exprss the following difference as the product of trignometric functions $\cos 7\theta - \cos\theta$.

x. In any triangle ΔABC, if $c = 16.1, \alpha = 42^0 45', \gamma = 74^0 32'$, then find "$\beta$" and "$\alpha$".

xi. Find the area of triangle ABC, given two sides and their included angle $a = 200, b = 120, \gamma = 150^0$.

xii. Find the solutions of the equation $\cot\theta = \frac{1}{\sqrt{3}}$ in the interval $[0, 2\pi]$.

xiii. Find the values of θ satisfying the equation $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$.

Section -II

Note: Attempt any three questions from the following. **10x3=30**

5. (a) Verify De Morgan's Laws for the given sets: $U = \{1, 2, 3, ...20\}, A = \{2, 4, 6, ...20\}, B = \{1, 3, 5, ...19\}$.

(b) Find the value of λ if A is singular matrix, $A = \begin{bmatrix} 4 & \lambda & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$.

6. (a) If the roots of $px^2 + qx + q = 0$ are α and β, then prove that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$.

(b) Resolve into partial fraction $\frac{x^4}{1 - x^4}$.

7. (a) The sum of an infinite geometric series is 9 and sum of square of its terms is $\frac{81}{5}$. Find the series.

(b) If $y = \frac{2}{5} + \frac{1.3}{2!}\left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!}\left(\frac{2}{5}\right)^3 + ...$, then prove that $y^2 + 2y - 4 = 0$.

8. (a) A railway train is running on a circular track of radius 500 meters at the rate of 30Km per hour. Through what angle will it turn in 10 sec?

(b) If $\tan\alpha = \frac{-15}{8}$ and $\sin\beta = \frac{-7}{25}$ and neither the terminal side of the angle of measure α nor that of β is in IV quadrant. Find $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$.

9. (a) One side of a triangular garden is 30m. If two corner angle are $22^0\frac{1}{2}$ and $112^0\frac{1}{2}$, find the cost of planting the grass at the rate of Rs.5 per square meter.

(b) Prove that $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$.