

Mathematics (Subjective)**SECTION-I****2. Write short answers of any eight parts from the following:**

(8×2=16)

- i. If $f(x) = 2x + 1$, then find $f \circ f(x)$.
- ii. Express the area A of a circle as a function of its circumference C .
- iii. Evaluate $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$
- iv. Define continuous function.
- v. Differentiate $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$ w.r.t x
- vi. Find $\frac{dy}{dx}$ if $y^2 - xy - x^2 + 4 = 0$
- vii. Differentiate $x^2 \sec 4x$ w.r.t x
- viii. Differentiate $\sin^2 x$ w.r.t. $\cos^4 x$
- ix. Find $f'(x)$ if $f(x) = e^x(1 + \ln x)$
- x. Find y_2 if $y = \ln(x^2 - 9)$
- xi. Prove that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$
- xii. Determine the interval in which $f(x) = \cos x$ is decreasing; $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

3. Write short answers of any eight parts from the following:

(8×2=16)

- i. Solve the differential equation $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$
- ii. Find the area between x -axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$
- iii. Evaluate: $\int_1^e x \ln x \, dx$
- iv. Evaluate the integral $\int \frac{-2x}{\sqrt{4-x^2}} \, dx$
- v. Evaluate: $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \, dx$
- vi. Evaluate the integral $\int (a - 2x)^{3/2} \, dx$
- vii. Find the approximate change in the volume of a cube if length of its each edge changes from 5 to 5.02.
- viii. Show that the points $A(0, 2)$, $B(\sqrt{3}, -1)$ and $C(0, -2)$ are vertices of a right triangle.
- ix. Convert the equation of line $4x + 7y - 2 = 0$ into normal form.
- x. Find the angle from the line with slope $\frac{-7}{3}$ to the line with slope $\frac{5}{2}$.
- xi. Find the pair of lines represented by $3x^2 + 7xy + 2y^2 = 0$.
- xii. Find the point of intersection of lines $3x + y + 12 = 0$ and $x + 2y - 1 = 0$.

4. Write short answers of any nine parts from the following:

(9×2=18)

- i. Define feasible region.
- ii. Graph the solution set of in-equality $3x + 7y \geq 21$.
- iii. Find equation of circle with ends of diameter at $(-3, 2)$ and $(5, -6)$.
- iv. Write down equation of tangent to the circle $x^2 + y^2 = 25$ at $(5 \cos \theta, 5 \sin \theta)$
- v. Find focus and vertex of Parabola $x^2 = 4(y - 1)$
- vi. Find equation of ellipse with data Foci $(\pm 3, 0)$ Minor axis of length 10.
- vii. Find center of hyperbola $x^2 - y^2 + 8x - 2y - 10 = 0$

- viii. Find equation of Normal to $y^2 = 4ax$ at $(at^2, 2at)$.
- ix. Find the sum of vector \overrightarrow{AB} and \overrightarrow{CD} given four points $A(1, -1)$, $B(2, 0)$, $C(-1, 3)$ and $D(-2, 2)$
- x. Find α , so that $|\alpha \underline{i} + (\alpha + 1)\underline{j} + 2\underline{k}| = 3$
- xii. If \underline{v} is a vector for which $\underline{v} \cdot \underline{i} = 0$, $\underline{v} \cdot \underline{j} = 0$, $\underline{v} \cdot \underline{k} = 0$, find \underline{v}
- xii. Find the area of triangle determined by the points $P(0, 0, 0)$, $Q(2, 3, 2)$ and $R(-1, 1, 4)$
- xiii. Find the value of $2\hat{i} \times 2\hat{j} \cdot \hat{k}$

SECTION-II

Note Attempt any three questions. Each question carries equal marks:

(10x3=30)

5. (a) Find the values of m and n , so that given function f is continuous at $x = 3$ when
- $$f(x) = \begin{cases} mx, & \text{if } x < 3 \\ n, & \text{if } x = 3 \\ -2x + 9, & \text{if } x > 3 \end{cases} \quad (05)$$
- (b) Find $\frac{dy}{dx}$, when $x = \frac{a(1-t^2)}{1+t^2}$, $y = \frac{2bt}{1+t^2}$ (05)
6. (a) If $y = (\cos^{-1}x)^2$, prove that $(1-x^2)y_2 - xy_1 - 2 = 0$. (05)
- (b) Evaluate the integral $\int e^x \sin x \cos x \, dx$. (05)
7. (a) Solve the differential equation $y - x \frac{dy}{dx} = 3 \left(1 + x \frac{dy}{dx}\right)$. (05)
- (b) Graph the feasible region and corner points of the inequalities (05)
- $$2x + y \leq 10; \quad x + 4y \leq 12; \quad x + 2y \leq 10;$$
8. (a) Show that the circles: $x^2 + y^2 + 2x - 8 = 0$; $x^2 + y^2 - 6x + 6y - 46 = 0$ touch internally. (05)
- (b) Using vector method, for any triangle ABC, prove that: $c^2 = a^2 + b^2 - 2ab \cos C$. (05)
9. (a) Find the focus, vertex and directrix of the Parabola; $x^2 = 4(y - 1)$ (05)
- (b) Find the lines represented by $3x^2 + 7xy + 2y^2 = 0$ and also find measure of the angle between them. (05)