

Mathematics (Subjective)

SECTION-I

2 Write short answers of any eight parts from the following: (8x2=16)

i. Define even function with example.

ii. Find $fog(x)$ if $f(x) = 2x + 1$, $g(x) = \frac{3}{x-1}$, $x \neq 1$.

iii. Evaluate: $\lim_{x \rightarrow 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$.

iv. Prove that $\operatorname{Sinh}2x = 2 \operatorname{Sinh}x \operatorname{Cosh}x$.

v. Find $\frac{dy}{dx}$ from first principles if $y = \frac{1}{\sqrt{x+a}}$.

vi. Differentiate w.r.t x ; $\frac{(x^2+1)^2}{x^2-1}$.

vii. Find $\frac{dy}{dx}$ if $x^2 - 4xy - 5y = 0$.

viii. Differentiate w.r.t θ ; $\tan^3\theta \sec^2\theta$.

ix. Find $f(x)$ if $f(x) = x^3 e^{1/x}$.

x. Find y_2 if $y = 2x^5 - 3x^4 + 4x^3 + x - 2$.

xi. Apply Maclaurin Series expansion to prove that

xii. Find extreme values for $f(x) = 3x^2$.

3 Write short answers of any eight parts from the following:

i. Evaluate $\int x\sqrt{x^2 - 1} dx$

ii. Use differentials to approximate the value of $(31)^{\frac{1}{5}}$

iii. Evaluate: $\int \frac{x}{\sqrt{4+x^2}} dx$

iv. Evaluate the integral $\int \frac{e^{m \tan^{-1} x}}{1+x^2} dx$

v. Evaluate: $\int_1^2 \frac{x}{x^2 + 2} dx$

vi. Find the area between $x - axis$ and the curve $y = 4x - x^2$

vii. Solve the differential equation $\frac{1}{x} \frac{dy}{dx} = \frac{1}{2}(1 + y^2)$

viii. The points $A(-5, -2)$ and $B(5, -4)$ are ends of a diameter of a circle. Find the centre and radius of circle.

ix. The coordinates of a point p are $(-6, 9)$. The axes are translated through the point $O(-3, 2)$. Find the coordinates of p referred to the new axes.

x. Check whether the origin and the point $p(5, -8)$ lies on the same side or on the opposite sides of the line $3x + 7y + 15 = 0$

xi. By means of slopes, show that the following points lie on the same line $(-4, 6); (3, 8); (10, 10)$.

xii. Determine the value of p such that the lines $2x - 3y - 1 = 0$, $3x - y - 5 = 0$ and $3x + py + 8 = 0$ meet at a point.

4 Write short answers of any nine parts from the following:

- i. Graph the solution set of $3y - 4 \leq 0$ in xy -plane.
- ii. Define convex region.
- iii. Find an equation of circle of radius a and lying in the second quadrant tangent to both the axes.
- iv. Find center and radius of circle $5x^2 + 5y^2 + 24x + 36y + 10 = 0$.
- v. Write down equation of normal to the circle $x^2 + y^2 = 25$ at $(4, 3)$.
- vi. Find vertex and directrix of the parabola $y^2 = -12x$.
- vii. Find the point of intersection of conics $x^2 + y^2 = 8$ and $x^2 - y^2 = 1$.
- viii. Find center and foci of hyperbola $\frac{y^2}{4} - x^2 = 1$.
- ix. Find a vector of magnitude 4 and is parallel to $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$.
- x. Find direction cosines of \overrightarrow{PQ} where $P = (2, 1, 5)$ and $Q = (1, 3, 1)$.
- xi. Find volume of parallelopiped whose edges are $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{v} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$ and $\mathbf{w} = \mathbf{j} + \mathbf{k}$
- xii. Find the value of $\begin{bmatrix} k & i & j \end{bmatrix}$.
- xiii. Find α so that $\mathbf{u} = \alpha \mathbf{i} + 2 \alpha \mathbf{j} - \mathbf{k}$ and $\mathbf{v} = \mathbf{i} + \alpha \mathbf{j} + 3\mathbf{k}$ are perpendicular.

SECTION-II

Note Attempt any three questions. Each question carries equal marks:

(10x3=30)

5. (a) Evaluate: $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$ (b) Differentiate $\cos \sqrt{x}$ from the first principle. (5+5)

6. (a) Show that $y = \frac{\ln x}{x}$ has maximum value at $x = e$ (b) Evaluate: $\int x^3 \cos x \, dx$ (5+5)

7. (a) Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x \, dx}{\sin x (2 + \sin x)} \, dx$ (b) Minimize $z = 2x + y$ subject to constraints
 $x + y \geq 3$ $7x + 5y \leq 35$
 $x \geq 0$ $y \geq 0$ (5+5)

8. (a) Find the coordinates of the points of intersection of the line $x + 2y = 6$ with the circle: $x^2 + y^2 - 2x - 2y - 39 = 0$ (5)
(b) If $\underline{a} = 4\underline{i} + 3\underline{j} + \underline{k}$ and $\underline{b} = 2\underline{i} - \underline{j} + 2\underline{k}$. Find a unit vector perpendicular to both \underline{a} and \underline{b} . Also find the sine of the angle between them. (5)

9. (a) Find the focus, vertex and directrix of the Parabola $x + 8 - y^2 + 2y = 0$ (5)
(b) Find coordinates of the circumcenter of the triangle whose vertices are $A(-2, 3)$, $B(-4, 1)$ and $C(3, 5)$. (5)

620-12-A