Roll No.__________ to be filled in by the candidate.

(For all sessions)

Mathematics (Essay Type)

Time: 2:30 Hours | Marks: 80

Section -I

2. Write short answers of any eight parts from the following. 2x8=16

i. If $f(x)=x^2-x$, find (a). $f(-2)$ (b). $f(x-1)$

ii. Find $\ell im_{x\to 2}\frac{x^3-8}{x^2+x-6}$.

iii. Find $\ell im_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta}$.

iv. Differentiate w.r.t "x". $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$.

v. Find $\frac{dy}{dx}$ if $3x+4y+7=0$.

vi. Differentiate w.r.t "x" $\cos\sqrt{x}+\sqrt{\sin x}$.

vii. Differentiate w.r.t "x" $\cot^{-1}\left(\frac{x}{a}\right)$.

viii. If $y=\ell og_{10}\left(ax^2+bx+c\right)$, then find $\frac{dy}{dx}$.

ix. If $y=x^2.\bar{e}^x$, then find $\frac{d^2y}{dx^2}$.

x. Apply Maclauren series, Prove that $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}+$ __________ ∞

xi. If $f(x)=\sqrt{x+1}$ and $g(x)=\frac{1}{x^2}$, then find (a). $(fog)(x)$ (b). $(gof)(x)$.

xii. Find the intervals in which $f(x)$ is increasing or decreasing $f(x)=\cos x,\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

3. Write short answers of any eight parts from the following. 2x8=16

i. Using differential find $\frac{dy}{dx}$, if $x^2+2y^2=16$.

ii. Evaluate $\int x\sqrt{x^2-1}\,dx$.

iii. Evaluate $\int\frac{\left(1-\sqrt{x}\right)^2}{\sqrt{x}}dx$.

iv. Evaluate $\int\sin^2 x\,dx$.

v. Evaluate $\int\frac{ax+b}{ax^2+2bx+c}dx$.

vi. Evaluate $\int e^{3x}\left(\frac{3\sin x-\cos x}{\sin^2 x}\right)dx$.

vii. Sove $\frac{dy}{dx}=\frac{y^2+1}{\bar{e}^x}$.

viii. Find an equation of the vertical line through (-5,3).

ix. Find an equation of the line through (-5,-3),(9,-1)

x. Convert $4x+7y-2=0$ in normal form.

xi. Find the area below the curve $y=3\sqrt{x}$ and above the x-axis between $x=1$ and $x=4$.

xii. Find the mid point of the line segment joining the points A(3,1) , B(-2,-4).

4. Write short answers of any nine parts from the following. 2x9=18

i. Graph the solution set by shading of inequality $5x-4y\le 20$.

ii. Find equation of circle with centre at $\left(\sqrt{2},-3\sqrt{3}\right)$ and radius $2\sqrt{2}$.

iii. Write equation of tangent to the circle $3x^2+3y^2+5x-13y+2=0$ at $\left(1,{}^{10}\!/_{3}\right)$.

iv. Find vertex of $x^2-4x-8y+4=0$.

v. Find point of intersection of conics $3x^2-4y^2=12$ and $3y^2-2x^2=7$.

vi. Find equation of parabola whose focus is F(-3,4) and directrix is $3x-4y+5=0$.

vii. Find the unit vector in the same direction of vector $\underline{V}=[3,-4]$.

viii. If $\overrightarrow{AB}=\overrightarrow{CD}$ find the co-ordinate of the point A when points B,C,D are (1,2)(-2,5) and (4,11) respectively

ix. Find $|3\underline{v}+\underline{w}|$ if $\underline{u}=\underline{i}+2\underline{j}-\underline{k}$, $\underline{v}=3\underline{i}-2\underline{j}+2\underline{k}$, $\underline{w}=5\underline{i}-\underline{j}+3\underline{k}$.

x. Find a vector of length 5 in the direction opposite that of $\underline{v}=\underline{i}-2\underline{j}+3\underline{k}$.

xi. Compute $\underline{b}\times\underline{a}$ if $\underline{b}=\underline{i}-\underline{j}+\underline{k}$, $\underline{a}=2\underline{i}+\underline{j}-\underline{k}$.

xii. Find the work done if the point at which the constant force $\overrightarrow{F}=4\underline{i}+3\underline{j}+5\underline{k}$ is applied to an object, moves from $p_1(3,1,-2)$ to $p_2(2,4,6)$.

xiii. If $\underline{a}+\underline{b}+\underline{c}=0$ then prove that $\underline{a}\times\underline{b}=\underline{b}\times\underline{c}=\underline{c}\times\underline{a}$.

Section -II

Note: Attempt any three questions from the following. **10x3=30**

5. (a) If $f(x)=\begin{cases} 3x-1 & if \quad x<1 \\ 4 & if \quad x=1 \\ 2x & if \quad x>1 \end{cases}$, then show $f(x)$ is continuous at $x=1$.

(b) If $x=\dfrac{a(1-t^2)}{1+t^2}$, $y=\dfrac{2bt}{1+t^2}$, then find $\dfrac{dy}{dx}$.

6. (a) Find the approximate increase in the volume of a cube of the length of its each edge changes from 5 to 5.02.

(b) Determine the value of P such that the lines $2x-3y-1=0, 3x-y-5=0$ and $3x+py+8=0$ meet at a point.

7. (a) Evaluate $\int_2^3 \left(x-\dfrac{1}{x}\right)^2 dx$.

(b) Minimize $z=2x+y$ subject to the constraints $x+y\geq 3$, $7x+5y\leq 35$, $x\geq 0$, $y\geq 0$.

8. (a) Write equations of two tangents from (2,3) to the circle $x^2+y^2=9$.

(b) Prove by vector method $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.

9. (a) Show that $\cos(x+h)=\cos x-h\sin x-\dfrac{h^2}{2!}\cos x+\dfrac{h^3}{3!}\sin x+$............

(b) Show that an equatin of the parabola with focus at $(a\cos\alpha, a\sin\alpha)$ and directrix $x\cos\alpha+y\sin\alpha+a=0$ is $(x\sin\alpha-y\cos\alpha)^2=4a(x\cos\alpha+y\sin\alpha)$.