

MATHEMATICS (Subjective) Group - I

Time: 02:30 Hours Marks: 80

SECTION - I

Solve any EIGHT parts:

16

Define irrational numbers.

Name the properties used in these equations: (a) $4 + 9 = 9 + 4$ (b) $1000 \times 1 = 1000$ i) Prove that $\bar{z} = z$ iff z is real.ii) Write two proper subsets of $\{a, b, c\}$

Define order of a set.

iii) Find the inverse of $\{(x, y) | y = 2x + 3, x \in \mathbb{R}\}$ iv) Find x and y if $\begin{bmatrix} x+3 & 1 \\ 3 & 3y-4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ v) If A and B are square matrices of the same order, then explain why in general $(A+B)(A-B) \neq A^2 - B^2$

vi) Define Hermitian matrix.

vii) Prove that $x^3 + y^3 = (x + y)(x + \omega y)(x + \omega^2 y)$ viii) If α, β are the roots of $x^2 + px + q = 0$, then prove that $(1 + \alpha)(1 + \beta) = 1 - c$

ix) Write two properties of the cube roots of unity.

Solve any EIGHT parts:

16

Define conditional equation.

If $\frac{2x+1}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3}$ find the value of B .Write partial fraction form of $\frac{8x^2}{(x^2+1)^2(1-x^2)}$ Find the 7th term of $1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \dots$ Find the number of terms in the A.P if $a_1 = 3, d = 7$ and $a_n = 59$ If 5 and 8 are two A.Ms between a and b . Find a and b .i) Find the 9th term of the harmonic sequence $\frac{-1}{5}, \frac{-1}{3}, -1, \dots$ ii) If the numbers $\frac{1}{k}, \frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in harmonic sequence, find k .

How many arrangements of the letters of the word, taken all together, can be made 'PAKPATTAN'.

Use mathematical induction to prove $1 + 3 + 5 + \dots + (2n-1) = n^2$ is true for $n = 1, n = 2$ Using binomial theorem find the value of $(1.03)^{\frac{1}{3}}$ upto three decimal places.iii) Use binomial theorem to expand $(a - \sqrt{2}x)^6$

Solve any NINE parts:

18

Define radian.

Convert $\frac{9\pi}{5}$ to sexagesimal system.Prove that $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$ Find the value of $\tan 15^\circ$, without using calculator.Prove that $\frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$

(Continued P. 2)

(vi) Prove that $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
 (vii) Find the period of $\cot 8x$
 (viii) State the law of sines.
 (ix) In the triangle ABC if $\alpha = 35^\circ 17'$, $\beta = 45^\circ 13'$ and $b = 421$. Find a.
 (x) Find the area of the triangle ABC if $a = 200$, $b = 120$, $\gamma = 150^\circ$
 (xi) Prove that $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} = \tan^{-1}\frac{9}{19}$
 (xii) Solve the equation $4\cos^2 x - 3 = 0$ where $x \in [0, 2\pi]$
 (xiii) Solve $\cosec\theta = 2$, where $\theta \in [0, 2\pi]$

SECTION - II Attempt any THREE questions. Each question carries 10 marks.

5. (a) Prove that the set $S = \{1, -1, i, -i\}$ is an abelian group under multiplication. 05
 (b) Obtain the sum of all integers in the first 1000 which are neither divisible by 5 nor by 2. 05

6. (a) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$ 05
 (b) A card is drawn from a deck of 52 playing cards. Find the probability that it is a diamond card or an ace. 05

7. (a) Find the values of a and b if -2 and 2 are the roots of the polynomial $x^3 - 4x^2 + ax + b$ 05
 (b) If $y = \frac{2}{5} + \frac{1.3}{2!} \left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!} \left(\frac{2}{5}\right)^3 + \dots$, then prove that $y^2 + 2y - 4 = 0$ 05

8. (a) Prove that $\frac{1}{\cosec\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\cosec\theta + \cot\theta}$ 05
 (b) If α, β, γ are angles of ΔABC , then prove that $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha \tan\beta \tan\gamma$ 05

9. (a) Prove that $(r_1 + r_2) \tan\frac{\gamma}{2} = c$ 05
 (b) Prove that $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1}3 = \frac{\pi}{4}$ 05