

MATHEMATICS (Subjective) Group – I

Time: 02:30 Hours Marks: 80

SECTION – I**tempt any EIGHT parts:**

16

Define irrational numbers.

Name the properties used in these equations: (a) $4 + 9 = 9 + 4$ (b) $1000 \times 1 = 1000$) Prove that $\bar{z} = z$ iff z is real.) Write two proper subsets of $\{a, b, c\}$

Define order of a set.

) Find the inverse of $\{(x, y) | y = 2x + 3, x \in \mathbb{R}\}$) Find x and y if $\begin{bmatrix} x+3 & 1 \\ 3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ i) If A and B are square matrices of the same order, then explain why in general $(A+B)(A-B) \neq A^2 - B^2$

) Define Hermitian matrix.

Prove that $x^3 + y^3 = (x+y)(x+\omega y)(x+\omega^2 y)$) If α, β are the roots of $x^2 + px + p + c = 0$, then prove that $(1+\alpha)(1+\beta) = 1+c$

) Write two properties of the cube roots of unity.

tempt any EIGHT parts:

16

Define conditional equation.

If $\frac{2x+1}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3}$ find the value of B .Write partial fraction form of $\frac{8x^2}{(x^2+1)^2(1-x^2)}$ Find the 7th term of $1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \dots$ Find the number of terms in the A.P if $a_1 = 3, d = 7$ and $a_n = 59$ If 5 and 8 are two A.Ms between a and b . Find a and b .) Find the 9th term of the harmonic sequence $\frac{-1}{5}, \frac{-1}{3}, -1, \dots$ i) If the numbers $\frac{1}{k}, \frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in harmonic sequence, find k .

How many arrangements of the letters of the word, taken all together, can be made 'PAKPAATTAN'.

Use mathematical induction to prove $1 + 3 + 5 + \dots + (2n-1) = n^2$ is true for $n=1, n=2$ Using binomial theorem find the value of $(1.03)^{\frac{1}{3}}$ upto three decimal places.) Use binomial theorem to expand $(a - \sqrt{2}x)^4$ **empt any NINE parts:**

18

Define radian.

Convert $\frac{9\pi}{5}$ to sexagesimal system.Prove that $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$ Find the value of $\tan 15^\circ$, without using calculator.Prove that $\frac{1-\cos\alpha}{\sin\alpha} = \tan \frac{\alpha}{2}$ **BLANK**

(Continued P 2)

- (vi) Prove that $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$
 (vii) Find the period of $\cot 8x$
 (viii) State the law of sines.
 (ix) In the triangle ABC if $\alpha = 35^\circ 17'$, $\beta = 45^\circ 13'$ and $b = 421$. Find a .
 (x) Find the area of the triangle ABC if $a = 200$, $b = 120$, $\gamma = 150^\circ$
 (xi) Prove that $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} = \tan^{-1} \frac{9}{19}$
 (xii) Solve the equation $4\cos^2 x - 3 = 0$ where $x \in [0, 2\pi]$
 (xiii) Solve $\operatorname{cosec} \theta = 2$, where $\theta \in [0, 2\pi]$

SECTION - II Attempt any THREE questions. Each question carries 10 marks.

5. (a) Prove that the set $S = \{1, -1, i, -i\}$ is an abelian group under multiplication. 05
 (b) Obtain the sum of all integers in the first 1000 which are neither divisible by 5 nor by 2. 05
6. (a) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$ 05
 (b) A card is drawn from a deck of 52 playing cards. Find the probability that it is a diamond card or an ace. 05
7. (a) Find the values of a and b if -2 and 2 are the roots of the polynomial $x^3 - 4x^2 + ax + b$ 05
 (b) If $y = \frac{2}{5} + \frac{1.3}{2!} \left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!} \left(\frac{2}{5}\right)^3 + \dots$, then prove that $y^2 + 2y - 4 = 0$ 05
8. (a) Prove that $\frac{1}{\operatorname{cosec} \theta - \cot \theta} - \frac{1}{\sin \theta} = \frac{1}{\sin \theta} - \frac{1}{\operatorname{cosec} \theta + \cot \theta}$ 05
 (b) If α, β, γ are angles of ΔABC , then prove that $\tan \alpha + \tan \beta + \tan \gamma = \tan \alpha \tan \beta \tan \gamma$ 05
9. (a) Prove that $(r_1 + r_2) \tan \frac{\gamma}{2} = c$ 05
 (b) Prove that $\sin^{-1} \frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{4}$ 05