MATHEMATICS (Subjective) Group – I

Time: 02:30 Hours Marks: 80

SECTION – I

2. **Attempt any EIGHT parts:** 16

(i) Does the set { 1 , – 1} possess closure property with respect to addition and subtraction?

(ii) Find the difference and product of the complex numbers (8 , 9) and (5 , – 6)

(iii) Find the multiplicative inverse of $(\sqrt{2}\ ,\ -\sqrt{5})$

(iv) If U = { 1, 2, 3, 4, 5, ... , 20 } and A = {1, 3, 5, ... , 19 } verify $A \cup A' = U$

(v) Write the inverse and contrapositive of the conditional $\sim q \rightarrow \sim p$

(vi) If a , b are elements of a group G then show that $(ab)^{-1} = b^{-1}a^{-1}$

(vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

(viii) If A and B are square matrices of the same order, then explain why in general $(A+B)(A-B) \neq A^2 - B^2$

(ix) Without expansion verify that $\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix} = 0$

(x) Use the remainder theorem to find remainder when first polynomial is divided by second polynomial $x^2 + 3x + 7\ ,\ x+1$

(xi) Find the condition that one root of the equation $x^2 + px + q = 0$ is multiplicative inverse of the other.

(xii) Discuss the nature of roots of the equation $2x^2 + 5x - 1 = 0$

3. **Attempt any EIGHT parts:** 16

(i) Resolve $\frac{3x-11}{(x^2+1)(x+3)}$ into partial fractions without finding constants.

(ii) Change $\frac{6x^3+5x^2-7}{2x^2-x-1}$ into proper rational fraction.

(iii) Find the indicated term of the sequence 1, – 3, 5, – 7, 9, – 11, ... , a_8

(iv) If $a_{n-3} = 2n - 5$ find the nth term of the sequence.

(v) Find the nth term of the geometric sequence if $\frac{a_5}{a_3} = \frac{4}{9}$ and $a_2 = \frac{4}{9}$

(vi) Find A, G, H and verify that A > G > H (G > 0) if a = 2 and b = 8.

(vii) Write $n(n-1)(n-2) \ldots . (n-r+1)$ in factorial form.

(viii) Find the value of n when ${}^{n}P_2 = 30$

(ix) A die is rolled. What is the probability that the dots on the top are greater than 4.

(x) Using binomial theorem expand $(a+2b)^5$.

(xi) Expand $(1-x)^{\frac{1}{2}}$ up to 4 terms.

(xii) Using binomial theorem to find the values of $\sqrt{99}$ to three places of decimals.

4. **Attempt any NINE parts:** 18

(i) Find ℓ, when $\theta = \pi$ radians , r = 6cm

(ii) Verify $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4} = 2$

(iii) Prove that $\sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$

(iv) Without using the table, find the value of sin(–300°)

(v) Prove that $\cos(\alpha + 45°) = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$

(vi) Prove that $\cot\alpha - \tan\alpha = 2\cot 2\alpha$

(vii) Find the period of $\tan\frac{x}{7}$

(viii) Find the greatest angle of triangle ABC if a = 16 , b = 20 , c = 33

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(ix) Find area of triangle ABC, given sides are a = 18 , b = 24 , c = 30

(x) Prove that $r_1\, r_2\, r_3 = rs^2$

(xi) Find the value of $\tan\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$

(xii) Find the solution of the equation $\sin x = \frac{-\sqrt{3}}{2}$ which lies in $[0\ ,\ 2\pi]$

(xiii) Solve $\cot\theta = \frac{1}{\sqrt{3}}$ where $\theta \in [0\ ,\ 2\pi]$

SECTION – II Attempt any THREE questions. Each question carries 10 marks.

5. (a) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)\,(a-b)\,(b-c)\,(c-a)$ 05

(b) If α , β are the roots of equation $ax^2+bx+c=0$ then form the equation whose roots are $\frac{1}{\alpha^3}, \frac{1}{\beta^3}$ 05

6. (a) Resolve into partial fractions: $\frac{9}{(x+2)^2(x-1)}$ 05

(b) If the H.M and A.M between two numbers are 4 and $\frac{9}{2}$ respectively, find the numbers. 05

7. (a) Prove that: ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^{n}C_r$ 05

(b) Determine the middle term in the expansion of $\left(\frac{1}{x} - \frac{x^2}{2}\right)^{12}$ 05

8. (a) If $\cot\theta = \frac{5}{2}$ and the terminal arm of the angle is in the I-quadrant, find the values of $\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$ 05

(b) Prove that $\frac{2\sin\theta\sin 2\theta}{\cos\theta + \cos 3\theta} = \tan 2\theta \tan\theta$ 05

9. (a) Show that $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2+b^2+c^2}{\Delta^2}$ 05

(b) Prove that $2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{12}{13}$ 05