1109-XI123

Intermediate Part First

Roll No. ___

MATHEMATICS (Subjective) Group – I

Time: 02:30 Hours Marks: 80

SECTION – I

2. **Attempt any EIGHT parts:** 1

(i) Express the complex number $1+i\sqrt{3}$ in polar form.

(ii) Whether closed or NOT with respect to addition and multiplication is { 1 } ?

(iii) If C = {a, b, c, d}, find P(C).

(iv) Let U = the set of English alphabet A = {x | x is a vowel}, B = {y | y is consonant}, verify DeMorgan's laws for these sets.

(v) Construct the truth table for statement $\sim(p \to q) \leftrightarrow (p \wedge \sim q)$

(vi) Find the matrix A if, $\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$

(vii) If $A = [a_{ij}]_{3\times 4}$, then show that $I_3A = A$

(viii) Without expansion verify that $\begin{vmatrix} mn & \ell & \ell^2 \\ n\ell & m & m^2 \\ \ell m & n & n^2 \end{vmatrix} = \begin{vmatrix} 1 & \ell^2 & \ell^3 \\ 1 & m^2 & m^3 \\ 1 & n^2 & n^3 \end{vmatrix}$

(ix) Find roots of equation by quadratic formula $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$

(x) Find four fourth roots of 625.

(xi) Find the condition that $\frac{a}{x-a}+\frac{b}{x-b}=5$ may have roots equal in magnitude but opposite in sign.

(xii) Show that roots of equation $px^2-(p-q)x-q=0$ will be rational.

3. **Attempt any EIGHT parts:** 16

(i) Define partial fraction resolution.

(ii) If $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in A.P, show that the common difference is $\frac{a-c}{2ac}$

(iii) Find A.M between $1-x+x^2$ and $1+x+x^2$.

(iv) How many terms of series $-7+(-5)+(-3)+\ .....$ amount to 65 ?

(v) Find vulgar fraction equivalent to the recurring decimal $1.\dot{3}\dot{4}$

(vi) If 5 is the H.M between 2 and b, find b.

(vii) How many arrangements of the letters of the word "MATHEMATICS", taking all together, can be made?

(viii) Find the value of n when ${}^{n}C_{10}=\frac{12\times 11}{2!}$

(ix) There are 5 green and 3 red balls in a box. One ball is taken out, find the probability that the ball taken out is red.

(x) Prove that $n!>2^{n-1}$ for $n=4,\ 5$.

(xi) Find 6th term in the expansion of $\left(x^2-\frac{3}{2x}\right)^{10}$.

(xii) Write first 4 terms of the expansion of $(8-2x)^{-1}$

4. **Attempt any NINE parts:** 18

(i) Find ℓ, when $\theta = 65°20'$, $r = 18\text{mm}$

(ii) Verify $\cos 2\theta = \cos^2\theta - \sin^2\theta$ when $\theta = 30°$, $45°$

(iii) Prove the identity $\frac{2\tan\theta}{1+\tan^2\theta}=2\sin\theta\cos\theta$

(iv) Prove that $\sin(180°+\alpha)\sin(90°-\alpha)=-\sin\alpha\cos\alpha$

(v) Prove that $\tan\left(\frac{\pi}{4}-\theta\right)+\tan\left(\frac{3\pi}{4}+\theta\right)=0$

(vi) Prove the identity $\frac{\sin 3\theta}{\sin\theta}-\frac{\cos 3\theta}{\cos\theta}=2$

(Continued P . . . . . 2)

(vii) Define period of a trigonometric function.

(viii) A vertical pole is 8m high and the length of its shadow is 6m. What is the angle of elevation of the sun at that moment?

(ix) Find the area of the triangle ABC, in which $b = 21.6$, $c = 30.2$ and $\alpha = 52°40'$

(x) Prove that with usual notations $r r_1 r_2 r_3 = \Delta^2$

(xi) Show that $\cos\left(2\sin^{-1}x\right) = 1 - 2x^2$

(xii) Solve the equation $\sin x + \cos x = 0$

(xiii) Solve the trigonometric equation $\operatorname{cosec}^2\theta = \frac{4}{3}$ in $[0, 2\pi]$

SECTION – II Attempt any THREE questions. Each question carries 10 marks.

5. (a) Solve the following system of linear equations by Cramer's rule: $\begin{array}{l} 2x_1 - x_2 + x_3 = 5 \\ 4x_1 + 2x_2 + 3x_3 = 8 \\ 3x_1 - 4x_2 - x_3 = 3 \end{array}$ 05

   (b) If α and β are the roots of $x^2 - 3x + 5 = 0$, form the equation whose roots are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$ 05

6. (a) Resolve into partial fractions: $\frac{x^4}{1-x^4}$ 05

   (b) Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^{n}C_r$ 05

7. (a) Obtain the sum of all integers in the first 1000 integers which are neither divisible by 5 nor by 2. 05

   (b) If $y = \frac{2}{5} + \frac{1.3}{2!}\left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!}\left(\frac{2}{5}\right)^3 + \ldots\ldots$, then show that $y^2 + 2y - 4 = 0$ 05

8. (a) Find the values of all trigonometric functions of $\frac{19\pi}{3}$ 05

   (b) Reduce $\sin^4\theta$ to an expression involving only functions of multiples of θ raised to the first power. 05

9. (a) Prove that $r = \frac{\Delta}{s}$ with usual notations. 05

   (b) Prove that $2\tan^{-1}\left(\frac{2}{3}\right) = \sin^{-1}\left(\frac{12}{13}\right)$ 05

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