Intermediate Part First Roll No. ________

MATHEMATICS (Subjective) Group – II

Time: 02:30 Hours Marks: 80

SECTION – I

Attempt any EIGHT parts: 16

i) Simplify by justifying each step: $\dfrac{4+16x}{4}$

ii) Find the multiplicative inverse of the complex number $\left(\sqrt{2}, -\sqrt{5}\right)$

iii) Prove that $\bar{z} = z$ if and only if z is real.

iv) Write any two proper subsets of the set $\{x \mid x \in Q \wedge 0 < x \le 2\}$

v) Write inverse and contrapositive of the conditional $q \rightarrow p$

vi) Define a semi-group.

vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

viii) If A and B are square matrices of the same order, then explain why in general $(A+B)(A-B) \neq A^2 - B^2$

ix) Define rank of a matrix.

x) Solve the equation; $x^3 + x^2 + x + 1 = 0$

xi) Discuss the nature of the roots of the equation: $2x^2 - 5x + 1 = 0$

xii) When $x^4 + 2x^3 + kx^2 + 3$ is divided by $x - 2$, the remainder is 1. Find the value of k.

Attempt any EIGHT parts: 16

i) Define an identity equation and give its example.

ii) Resolve into partial fractions: $\dfrac{1}{x^2-1}$

iii) Write in mixed form: $\dfrac{6x^3+5x^2-7}{2x^2-x-1}$

iv) If $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in A.P. Show that common difference is $\dfrac{a-c}{2ac}$

v) Find the sum of 20 terms of the series, whose rth term is $3r + 1$

vi) If x and y are positive distinct real numbers, show that G.M between x and y is less than A.M.

vii) If $y = \dfrac{x}{2} + \dfrac{x^2}{4} + \dfrac{x^3}{8} + \ldots\ldots$, $0 < x < 2$, prove that $x = \dfrac{2y}{1+y}$

viii) Find the 12th term of harmonic sequence $\dfrac{1}{3}, \dfrac{2}{9}, \dfrac{1}{6}, \ldots\ldots$

ix) Express in factorial form: $\dfrac{(n+1)(n)(n-1)}{3 \cdot 2 \cdot 1}$

x) Prove that $n! > 2^n - 1$ is true for $n = 5$, $n = 6$

xi) Using binomial theorem find the value of $(1.03)^{\frac{1}{3}}$ upto three decimal places.

xii) Use binomial series to find $(1.03)^{\frac{1}{3}}$ upto three places of decimals.

Attempt any NINE parts: 18

i) Convert $54°45'$ into radians.

ii) Evaluate $\dfrac{\tan\frac{\pi}{3} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{6}}$

iii) Prove that $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

iv) Prove that $\tan\left(\dfrac{\pi}{4} - \theta\right) + \tan\left(\dfrac{3\pi}{4} + \theta\right) = 0$

v) If α, β, γ are angles of a triangle ABC then prove that $\tan(\alpha + \beta) + \tan\gamma = 0$

(Continued P . . . . . 2)

(vi) Prove that $\frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$

(vii) Find the period of tan 4x

(viii) State the law of cosines (any two).

(ix) At the top of a cliff 80 meters high the angle of depression of a boat is 12°. How far is the boat from the cliff ?

(x) Define angle of elevation.

(xi) Show that $\sin(2\cos^{-1}x) = 2x\sqrt{1-x^2}$

(xii) Find solution of equation sec x = -2 which lie in $[0, 2\pi]$

(xiii) Solve the equation $1 + \cos x = 0$

SECTION – II Attempt any THREE questions. Each question carries 10 marks.

(a) If (G , ✻) is a group and $a \in G$, then show that inverse of a is unique in G. 05

(b) If ℓ, m, n are the pth , qth and rth terms of an A.P. Show that $p(m-n)+q(n-\ell)+r(\ell-m)=0$ 05

(a) Solve the given system of equations by Cramer's rule: 05

$$\begin{aligned} 2x + 2y + z &= 3 \\ 3x - 2y - 2z &= 1 \\ 5x + y - 3z &= 2 \end{aligned}$$

(b) Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6 ? 05

(a) Show that the roots of $x^2+(mx+c)^2=a^2$ will be equal if $c^2=a^2(1+m^2)$ 05

(b) Find the term in the expansion of $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11}$ involving x^5 05

(a) If $\tan\theta=\frac{1}{\sqrt{7}}$ and the terminal arm of the angle is not in the III quad. Find the value of $\frac{\csc^2\theta-\sec^2\theta}{\csc^2\theta+\sec^2\theta}$ 05

(b) Without using calculator show that $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$ 05

(a) Prove that $\Delta = 4R\,r\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$ 05

(b) Prove that $\cos^{-1}\frac{63}{65}+2\tan^{-1}\frac{1}{5}=\sin^{-1}\frac{3}{5}$ 05