Intermediate Part First

Roll No. ________

MATHEMATICS (Subjective) Group – II

Time: 02:30 Hours Marks: 80

SECTION – I

2. Attempt any EIGHT parts: 16

(i) Simplify $(-1)^{\frac{-21}{2}}$

(ii) Show that $z\bar{z} = |z|^2$

(iii) Find multiplicative inverse of $-3-5i$

(iv) Find converse and inverse of $\sim p \rightarrow \sim q$

(v) Write $\{x \mid x \in 0 \wedge 3 < x < 12\}$ in descriptive and tabular form.

(vi) Show that subtraction is non-commutative on 'N'.

(vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

(viii) Find inverse of $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

(ix) Without expansion show that $\begin{vmatrix} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} = 0$

(x) Evaluate $(1+\omega-\omega^2)^8$

(xi) When the polynomial x^3+2x^2+kx+4 is divided by $x-2$, the remainder is 14. Find value of k.

(xii) If α, β are the roots of $3x^2-2x+4=0$, then find the value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

3. Attempt any EIGHT parts: 16

(i) Write only partial fraction form of $\frac{x^2+1}{x^3+1}$ without finding constants.

(ii) Resolve $\frac{7x+5}{(x+3)(x+4)}$ into partial fraction.

(iii) Find the 13th term of the sequence $x, 1, 2-x, 3-2x, \ldots\ldots$

(iv) Show that reciprocals of the terms of the geometric sequence $a_1, a_1r^2, a_1r^4, \ldots\ldots$ form another geometric sequence.

(v) If $y = 1+\frac{x}{2}+\frac{x^2}{4}+\ldots\ldots$ show that $x = 2\left(\frac{y-1}{y}\right)$

(vi) Find the nth term of H.P. $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \ldots\ldots$

(vii) Write $\frac{(n+1)(n)(n-1)}{3\cdot2\cdot1}$ in the factorial form.

(viii) Find the value of n when ${}^{n}P_4 : {}^{n-1}P_3 = 9:1$

(ix) Find the number of diagonals of a 6-sided figure.

(x) Prove the formula $1+2+4+\ldots\ldots+2^{n-1} = 2^n-1$ for $n=1, 2$

(xi) Using binomial theorem, expand $(a+2b)^5$

(xii) Expand $(2-3x)^{-2}$ upto 4-terms.

4. Attempt any NINE parts: 18

(i) Find θ when $\ell = 1.5$cm ; $r = 2.5$cm

(ii) Verify $\sin 60° \cos 30° - \cos 60° \sin 30° = \sin 30°$

(iii) Prove that $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

(iv) If α, β and γ are the angles of triangle ABC then prove that $\sin(\alpha+\beta) = \sin(\gamma)$

(v) Prove that $\cos(\alpha+45°) = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$

(Continued P . . . . . 2)

(vi) Express $2\sin(3\theta)\cos\theta$ as sum or difference.

(vii) Find the period of $\tan\left(\frac{x}{7}\right)$

(viii) Find the value of $\sin 53°40'$

(ix) Find area of triangle ABC, if $a = 200$; $b = 120$ and $\gamma = 150°$

(x) Find the value of α if $a = 7$; $b = 7$, $c = 9$

(xi) Show that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$

(xii) Solve the equation $\sin x = \frac{1}{2}$

(xiii) Find the solutions of $\cot\theta = \frac{1}{\sqrt{3}}$; $\theta \in [0, 2\pi]$

SECTION – II   Attempt any THREE questions. Each question carries 10 marks.

5. (a) Show that $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$ 05

(b) Prove that $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$ will have equal roots if $c^2 = a^2m^2 + b^2$ 05

6. (a) Resolve into partial fraction: $\frac{1}{(x-1)(2x-1)(3x-1)}$ 05

(b) For what value of n, $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the positive geometric mean between a and b. 05

7. (a) Prove that: ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^{n}C_r$ 05

(b) If $y = \frac{1}{3} + \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \ldots\ldots$ then prove that $y^2 + 2y - 2 = 0$ 05

8. (a) If $\operatorname{cosec}\theta = \frac{m^2+1}{2m}$ and $m > 0\left(0 < \theta < \frac{\pi}{2}\right)$ find the values of the remaining trigonometric ratios. 05

(b) Prove without using calculator $\sin 19° \cos 11° + \sin 71° \sin 11° = \frac{1}{2}$ 05

9. (a) Show that $r_3 = 4R\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\sin\frac{\gamma}{2}$ 05

(b) Prove that $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}$ 05