Intermediate Part First

Roll No. _____

MATHEMATICS (Subjective) Group – II

Time: 02:30 Hours Marks: 80

SECTION – I

2. **Attempt any EIGHT parts:** 16

(i) State trichotomy property of real numbers.

(ii) Express $\frac{i}{1+i}$ in the form of a + bi

(iii) Write the descriptive and tabular form of set $A = \{x : x \in E \wedge 4 \le x \le 10\}$

(iv) Find the converse and inverse of $q \rightarrow p$

(v) Define group.

(vi) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

(vii) If A and B are non-singular matrices, then show that $(AB)^{-1} = B^{-1}A^{-1}$

(viii) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$, show that $A + A^t$ is symmetric.

(ix) Find three cube roots of unity.

(x) Show that $x + a$ is a factor of $x^n + a^n$ where n is odd integer.

(xi) If α and β are roots of $5x^2 - x - 2 = 0$, then find value of $\frac{3}{\alpha} + \frac{3}{\beta}$.

(xii) Show that roots of equation $(p+q)x^2 - px - q = 0$ are rational.

3. **Attempt any EIGHT parts:** 16

(i) $\frac{3x^2+1}{x-2}$ is an improper fraction, convert into proper fraction.

(ii) Find a_8 for the sequence 1, 1, –3, 5, –7, 9, ...

(iii) Sum the series $(-3) + (-1) + 1 + 3 + \ldots\ldots + a_{16}$

(iv) Find the nth term of H.P. $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}$ .....

(v) Find the sum of infinite geometric series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots$

(vi) Define arithmetic progression.

(vii) Evaluate $\frac{8!}{6!}$

(viii) How many signals can be made with 4-different flags when any number of them are to be used at a time?

(ix) Find the value of n when ${}^nC_5 = {}^nC_4$

(x) Expand $\left(\frac{a}{2} - \frac{2}{a}\right)^6$

(xi) Find the term independent of x in the expansion of $\left(\frac{x}{2} + \frac{2}{x^2}\right)^{12}$

(xii) Using binomial theorem, find the value of $\sqrt{99}$ up to three places of decimals.

4. **Attempt any NINE parts:** 18

(i) Find θ, when $\ell = 3.2m$, $r = 2m$

(ii) Prove the identity $\cot^4\theta + \cot^2\theta = \csc^4\theta - \csc^2\theta$

(iii) For $\theta = \frac{-71}{6}\pi$, find the values of $\sin\theta$ and $\cos\theta$

(iv) Show that $\cos 330° \sin 600° + \cos 120° \sin 150° = -1$

(v) Prove that $\sin 2\alpha = 2 \sin\alpha \cos\alpha$

(vi) Express $2 \sin 7\theta \sin 2\theta$ as a sum or difference.

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(vii) Find the period of $\sin\frac{x}{5}$

(viii) In the triangle ABC if $a = 36.21$, $c = 30.14$, $\beta = 78°10'$ find angle γ

(ix) The area of triangle is 2437. If $a = 79$, $c = 97$ find angle β .

(x) Show that $r_2 = s\tan\frac{\beta}{2}$

(xi) Without using table / calculator show that $\cos^{-1}\frac{4}{5} = \cot^{-1}\frac{4}{3}$

(xii) Find the solutions of $\operatorname{cosec} x = 2$, $x \in [0, 2\pi]$

(xiii) Solve $\sin x + \cos x = 0$

SECTION – II Attempt any THREE questions. Each question carries 10 marks.

5. (a) Solve the system of linear equations by Cramer's rule: 05

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 8 \\ x_1 + 2x_2 + 2x_3 &= 6 \\ x_1 - 2x_2 - x_3 &= 1 \end{aligned}$$

(b) Solve the equation: $x^2 - \frac{x}{2} - 7 = x - 3\sqrt{2x^2 - 3x + 2}$ 05

6. (a) Resolve into partial fractions: $\frac{2x^4}{(x-3)(x+2)^2}$ 05

(b) Find value of n and r when ${}^{n-1}C_{r-1} : {}^{n}C_r : {}^{n+1}C_{r+1} = 3:6:11$ 05

7. (a) The sum of three numbers in A.P is 24 and their product is 440. Find the numbers. 05

(b) If x is so small that its square and higher powers can be neglected, then show that:

$$\frac{(1+x)^{\frac{1}{2}}(4-3x)^{\frac{3}{2}}}{(8+5x)^{\frac{1}{3}}} \approx 4\left(1 - \frac{5x}{6}\right)$$ 05

8. (a) Prove the identity $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{2}{1 - 2\sin^2\theta}$ 05

(b) Prove that $\frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$ 05

9. (a) Prove that $r_3 = s\tan\frac{\gamma}{2}$ 05

(b) Prove that $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$ 05