Intermediate Part Second Roll No. ________

MATHEMATICS (Subjective) Group – I

Time: 02:30 Hours Marks: 80

SECTION – I

2. **Attempt any EIGHT parts:** 16

(i) Define exponential function.

(ii) $f(x) = 2x + 1$, $g(x) = x^2 - 1$, find $g(f(x))$

(iii) Prove the identity $\cosh^2 x + \sinh^2 x = \cosh 2x$

(iv) Find by definition derivative of $\frac{1}{x-a}$

(v) Differentiate $\frac{(x^2+1)^2}{x^2-1}$ w.r.t. x

(vi) Find $\frac{dy}{dx}$ by making suitable substitution if $y = \sqrt{x + \sqrt{x}}$

(vii) Prove that $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

(viii) Differentiate $\sin^2 x$ w.r.t. $\cos^4 x$

(ix) Find $\frac{dy}{dx}$ if $y = e^{-2x} \sin 2x$

(x) Find y_2 if $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

(xi) Apply the Maclaurin series, prove $e^{2x} = 1 + 2x + 2x^2 + \ldots\ldots$

(xii) Determine the interval in which f is increasing or decreasing if $f(x) = 4 - x^2$, $x \in (-2, 2)$

3. **Attempt any EIGHT parts:** 16

(i) Find δy and dy of function $f(x) = x^2$ when $x = 2$ and $dx = 0.01$

(ii) Using differential find $\frac{dy}{dx}$ if $xy - \ell n x = c$

(iii) Evaluate $\int (x+1)(x-3)\, dx$

(iv) Evaluate $\int \frac{1}{\sqrt{x}\,(\sqrt{x}+1)}\, dx$

(v) Evaluate $\int \frac{1}{1+\cos x}\, dx$, $\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$

(vi) Evaluate $\int \frac{x^2}{4+x^2}\, dx$

(vii) Evaluate $\int x \ell n x\, dx$

(viii) Evaluate $\int x \sin x\, dx$

(ix) Evaluate $\int_{-1}^{3} (x^3 + 3x^2)\, dx$

(x) Evaluate $\int_{0}^{3} \frac{dx}{x^2+9}$

(xi) Define objective function.

(xii) Graph the solution set of linear inequality $2x + y \leq 6$

(Continued P/2)

Attempt any NINE parts: 18

(i) Find the point trisecting the join of A (− 1 , 4) and B (6 , 2)

(ii) Find an equation of the line through A (− 6 , 5) having slope 7

(iii) Find the point of intersection of the lines $x - 2y + 1 = 0$ and $2x - y + 2 = 0$

(iv) Define the homogeneous equation.

(v) Find the radius of the circle $x^2 + y^2 - 6x + 4y + 13 = 0$

(vi) Find the equation of axis and focus of parabola $x^2 = -16y$

(vii) Find the foci of the ellipse $25x^2 + 9y^2 = 225$

(viii) Find the equations of directrices of hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(ix) Find the vector from point A to the origin where $\overline{AB} = 4\underline{i} - 2\underline{j}$ and B is the point (− 2 , 5)

(x) Define the direction cosines of a vector.

(xi) Find a unit vector in the direction of $\bar{V} = \underline{i} + 2\underline{j} - \underline{k}$

(xii) Find a scalar 'α' so that the vectors $2\underline{i} + \alpha\underline{j} + 5\underline{k}$ and $3\underline{i} + \underline{j} + \alpha\underline{k}$ are perpendicular.

(xiii) If $\bar{a} + \bar{b} + \bar{c} = 0$ then prove that $\bar{a} \times \bar{b} = \bar{b} \times \bar{c} = \bar{c} \times \bar{a}$

SECTION – II   Attempt any THREE questions. Each question carries 10 marks.

. (a) Find m and n so that the given function f is continuous at x = 3 $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$ 05

(b) If $y = (\cos^{-1} x)^2$, prove that $(1 - x^2)y_2 - xy_1 - 2 = 0$ 05

. (a) Evaluate $\int \frac{x - 2}{(x+1)(x^2+1)} dx$ 05

(b) The average entry test score of engineering candidates was 592 in the year 1998 while the score was 564 in 2002. Assuming that the relationship between time and score is linear, find the average score for 2006. 05

. (a) Find the area bounded by curve $y = x^3 - 4x$ and the x-axis. 05

(b) Maximize : $f(x, y) = 2x + 5y$ subject to
Constraints : $2y - x \le 8$, $x - y \le 4$, $x \ge 0$, $y \ge 0$ 05

(a) The vertices of a triangle are A (−2 , 3) , B (−4 , 1) and C (3 , 5). Find coordinates of the orthocenter of the triangle. 05

(b) Show that the lines $3x - 2y = 0$ and $2x + 3y - 13 = 0$ are tangents to the circle $x^2 + y^2 + 6x - 4y = 0$ 05

(a) Find the equations of tangent and normal to the conic $\frac{x^2}{8} + \frac{y^2}{9} = 1$ at the point $\left(\frac{8}{3}, 1\right)$ 05

(b) Prove that $\text{Cos}(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$ 05