Intermediate Part Second Roll No. ________

MATHEMATICS (Subjective) Group – I

Time: 02:30 Hours Marks: 80

SECTION – I

2. Attempt any EIGHT parts: 16

(i) Find the domain and range of $g(x)=\sqrt{x^2-4}$

(ii) Find $f^{-1}(x)$ if $f(x)=\dfrac{2x+1}{x-1}$

(iii) Find $\lim\limits_{x\to a}\dfrac{x^n-a^n}{x^m-a^m}$

(iv) Find $\lim\limits_{x\to 0}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$, $x>0$

(v) If $y=x^4+2x^2+2$, prove that $\dfrac{dy}{dx}=4x\sqrt{y-1}$

(vi) Differentiate $\sin x$ w.r.t. $\cot x$

(vii) Find $\dfrac{dy}{dx}$ if $x^2-4xy-5y=0$

(viii) If $f(x)=\ell n\sqrt{e^{2x}+e^{-2x}}$, find $f'(x)$

(ix) If $y=\ell n(\tanh x)$, find $\dfrac{dy}{dx}$

(x) If $y=x^2\,\ell n\left(\dfrac{1}{x}\right)$, find $\dfrac{dy}{dx}$

(xi) If $x=a(\cos t+\sin t)$, $y=a(\sin t-t\cos t)$, find $\dfrac{dy}{dx}$

(xii) Apply Maclaurin series prove that $e^{2x}=1+2x+4\dfrac{x^2}{2!}+\ldots\ldots$

3. Attempt any EIGHT parts: 16

(i) Use differential to approximate the value of $\sqrt{17}$

(ii) Evaluate $\int x\sqrt{x^2-1}\,dx$

(iii) Evaluate $\int\left(\sqrt{x}+\dfrac{1}{\sqrt{x}}\right)dx$

(iv) Evaluate $\int\tan^2 x\,dx$

(v) Evaluate $\int\dfrac{\sec^2 x}{\sqrt{\tan x}}\,dx$

(vi) Evaluate $\int \ell nx\,dx$

(vii) Evaluate $\int_0^{\frac{\pi}{6}} x\cos x\,dx$

(viii) Solve the differential equation $y\,dx+x\,dy=0$

(ix) Find the coordinates of the point that divides the join of $A(-6,3)$ and $B(5,-2)$ in the ratio 2 : 3 internally.

(x) By means of slopes that the points (4, – 5) , (7 , 5) and (10 , 15) lie on the same line.

(xi) Fi... e with y-intercept: –7 and slope: –5.

... y $2x^2+3xy-5y^2=0$

(Continued P/2)

4. **Attempt any NINE parts:** 18
   (i) Graph the solution set of $3x - 2y \geq 6$
   (ii) Find equation of circle with center at $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$
   (iii) Find length of tangent from point P (–5, 10) to circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
   (iv) Find vertex and directrix of parabola $x^2 = -16y$
   (v) Find equation of parabola with focus (– 3 , 1) and directrix x = 3
   (vi) Find center and foci of $\frac{x^2}{4} - \frac{y^2}{9} = 1$
   (vii) Find eccentricity and vertex of $\frac{y^2}{16} - \frac{x^2}{9} = 1$
   (viii) Write the vector $\overrightarrow{PQ}$ in the form $x\underline{i} + y\underline{j}$, P(2 , 3) , Q(6 ,–2)
   (ix) Find a unit vector in the direction of $\underline{v} = 2\underline{i} - \underline{j}$
   (x) Find a vector whose magnitude is 4 and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$
   (xi) Find a real number α so that $\underline{u} = \alpha\underline{i} + 2\alpha\underline{j} - \underline{k}$, $\underline{v} = \underline{i} + \alpha\underline{j} + 3\underline{k}$ are perpendicular.
   (xii) Compute $\underline{b} \times \underline{a}$ if $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$, $\underline{b} = \underline{i} - \underline{j} + \underline{k}$
   (xiii) Prove that $\underline{u} \cdot (\underline{v} \times \underline{w}) + \underline{v} \cdot (\underline{w} \times \underline{u}) + \underline{w} \cdot (\underline{u} \times \underline{v}) = 3\underline{u} \cdot (\underline{v} \times \underline{w})$

SECTION – II Attempt any THREE questions. Each question carries 10 marks.

5. (a) Evaluate $\lim_{\theta \to 0} \frac{\tan\theta - \sin\theta}{\sin^3\theta}$ 05
   (b) Show that $\frac{dy}{dx} = \frac{y}{x}$ if $\frac{y}{x} = \tan^{-1}\frac{x}{y}$ 05

6. (a) Solve $\int e^{-x} \sin 2x \, dx$ 05
   (b) Find the angles of the triangle whose vertices are A(–5 , 4) , B(–2 , –1) and C(7 , –5) 05

7. (a) Evaluate $\int_0^{\frac{\pi}{4}} \cos^4 t \, dt$ 05
   (b) Maximize f (x , y) = x + 3y subject to the constraints: $2x + 5y \leq 30$; $5x + 4y \leq 20$; $x \geq 0$, $y \geq 0$ 05

8. (a) Write an equation of circle passing through the points A(–7 , 7) , B(5 , –1) , C(10 , 0) 05
   (b) Given force $\vec{F} = 2i + j - 3k$ acting at a point A(1, – 2, 1) find the moment of $\vec{F}$ about the point B(2 , 0, – 2) 05

9. (a) Show that $y = \frac{\ell nx}{x}$ has maximum value at x = e 05
   (b) Show that the ordinate at any point P of the parabola is a mean proportional between the length of the latus rectum and the abscissa of P. 05