

MATHEMATICS (Subjective) Group – I

Time: 02:30 Hours

Marks: 80

SECTION – I**2. Attempt any EIGHT parts:**

16

- (i) Prove that $\operatorname{sech}^2 x = 1 - \tanh^2 x$
- (ii) Find $f^{-1}(x)$ if $f(x) = (-x + 9)^3$
- (iii) Evaluate $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x}$
- (iv) Express the perimeter P of square as a function of its area A .
- (v) Evaluate $\lim_{x \rightarrow 3} \frac{x - 3}{\sqrt{x} - \sqrt{3}}$
- (vi) Find the derivative of $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2$ with respect to x
- (vii) If $y = \sqrt{x + \sqrt{x}}$, then find $\frac{dy}{dx}$
- (viii) Differentiate $\sin x$ w.r.t. $\cot x$
- (ix) Find $\frac{dy}{dx}$ if $y = x e^{\sin x}$
- (x) If $f(x) = e^x(1 + \ln x)$, find $f'(x)$
- (xi) Determine the intervals in which $f(x) = x^2 + 3x + 2$, $x \in (-4, 1)$ is increasing.
- (xii) If $y = \tanh(x^2)$, then find $\frac{dy}{dx}$

3. Attempt any EIGHT parts:

16

- (i) Using differentials find $\frac{dy}{dx}$; $x^2 + 2y^2 = 16$
- (ii) Evaluate $\int (\sqrt{x} + 1)^2 dx$
- (iii) Evaluate $\int \frac{dx}{(x^2 + 4x + 13)}$
- (iv) Evaluate $\int x^2 \sin x dx$
- (v) Evaluate $\int e^{2x} [-\sin x + 2 \cos x] dx$
- (vi) Evaluate $\int \frac{3x + 1}{x^2 - x + 6} dx$
- (vii) Evaluate $\int_{-1}^1 (x^{\frac{1}{3}} + 1) dx$
- (viii) Find the area between the x -axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$
- (ix) Find the points trisecting the join of $A(-1, 4)$ and $B(6, 2)$.
- (x) Find an equation of the horizontal line through $(7, -9)$.
- (xi) Convert $15y - 8x + 3 = 0$ in normal form.
- (xii) Find the lines represented by $10x^2 - 23xy - 5y^2 = 0$

4. Attempt any NINE parts:

18

- (i) Define the optimal solution.
- (ii) Indicate the solution set by shading of $2x + y \leq 6$
- (iii) Find an equation of the circle with ends of a diameter at $(-3, 2)$ and $(5, -6)$
- (iv) Check the position of the point $(5, 6)$ with respect to the circle $x^2 + y^2 = 81$
- (v) Write an equation of parabola with given elements : Directrix $x = -2$, Focus $(2, 2)$
- (vi) Form the equation of ellipse from center $(0, 0)$; focus $(0, -3)$, vertex $(0, 4)$
- (vii) Investigate the center and foci of $\frac{y^2}{16} - \frac{x^2}{9} = 1$

(Continued P/2)

- (viii) If O is the origin and $\overrightarrow{OP} = \overrightarrow{AB}$, find the point P when A and B are $(-3, 7)$ and $(1, 0)$
- (ix) Find α , so that $|\alpha \underline{i} + (\alpha + 1)\underline{j} + 2\underline{k}| = 3$
- (x) Show that the vectors $2\underline{i} - \underline{j} + \underline{k}$, $\underline{i} - 3\underline{j} - 5\underline{k}$ and $3\underline{i} - 4\underline{j} - 4\underline{k}$ form the sides of a right triangle.
- (xi) Calculate area of the parallelogram whose vertices are P(0, 0, 0), Q(-1, 2, 4), R(2, -1, 4) and S(1, 1, 8)
- (xii) Prove that A(-3, 5, -4), B(-1, 1, 1), C(-1, 2, 2) and D(-3, 4, -5) are coplanar.
- (xiii) Give a force $\underline{F} = 2\underline{i} + \underline{j} - 3\underline{k}$ acting at a point A(1, -2, 1). Find the moment of \underline{F} about the points B(2, 0, -2)

SECTION – II Attempt any THREE questions. Each question carries 10 marks.

5. (a) For the real valued function $f(x) = (-x + 9)^3$ find
 (i) $f^{-1}(x)$ (ii) $f^{-1}(-1)$ and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ 05
- (b) If $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ 05
6. (a) Evaluate: $\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$ 05
- (b) Find h such that the points A(h, 1), B(2, 7) and C(-6, -7) are the vertices of right triangle with right angle at the vertex A. 05
7. (a) Evaluate $\int_0^{\frac{\pi}{4}} \cos^4 t \, dt$ 05
- (b) Graph the feasible region of the linear inequality and also find corner points:
 $x + y \leq 5$, $-2x + y \leq 2$, $x \geq 0$, $y \geq 0$ 05
8. (a) Find an equation of the line through the intersection of the lines $x + 2y + 3 = 0$, $3x + 4y + 7 = 0$ and making equal intercepts on the axes. 05
- (b) Find an equation of the circle passing through A(-3, 1) with radius 2 and center at $2x - 3y + 3 = 0$ 05
9. (a) Find foci, center, vertices and directrices of hyperbola $4y^2 + 12y - x^2 + 4x + 1 = 0$ 05
- (b) Find a unit vector perpendicular to plane containing \vec{a} and \vec{b} . Also find the sine of angle between them. $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$ 05