MATHEMATICS (Subjective) Group – I

Time: 02:30 Hours Marks: 80

SECTION – I

2. **Attempt any EIGHT parts:** 16

(i) Define constant function with example.

(ii) Find $f^{-1}(x)$ if $f(x) = -2x + 8$

(iii) Evaluate $\lim_{x \to 1} \frac{x^3 - 3x^2 + 3x - 1}{x^3 - x}$

(iv) Find derivative by definition $\frac{1}{x^{40}}$

(v) Differentiate w.r.t. x, $\frac{\sqrt{1+x}}{\sqrt{1-x}}$

(vi) Find $\frac{dy}{dx}$, $xy + y^2 = 2$

(vii) Differentiate w.r.t. x, $\cos\sqrt{x} + \sqrt{\sin x}$

(viii) Differentiate $\cos^{-1}\left(\frac{x}{a}\right)$

(ix) Find $\frac{dy}{dx}$ if $y = x\, e^{\sin x}$

(x) Determine the interval in which $f(x) = \cos x$ is increasing or decreasing for the domain $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(xi) Define problem constraint.

(xii) Graph the solution set of $5x - 4y \leq 20$

3. **Attempt any EIGHT parts:** 16

(i) Find the area bounded by cos, function from $x = \frac{-\pi}{2}$ to $x = \frac{\pi}{2}$.

(ii) Solve the differential equation $(e^x + e^{-x})\frac{dy}{dx} = e^x - e^{-x}$

(iii) Evaluate $\int_{-1}^{2} (x + |x|)\, dx$

(iv) Evaluate $\int \frac{e^{m \tan^{-1} x}}{1 + x^2}\, dx$

(v) Evaluate $\int \frac{1}{x \ell n x}\, dx$

(vi) Evaluate $\int \frac{(1 - \sqrt{x})^2}{\sqrt{x}}\, dx$

(vii) Use differential find $\frac{dy}{dx}$ if $x^2 + 2y^2 = 16$.

(viii) Find the value of $2\underline{i} \times 2\underline{j} \cdot \underline{k}$

(ix) Find a unit vector in the direction of $\underline{v} = \underline{i} + 2\underline{j} - \underline{k}$

(x) If $\overline{AB} = \overline{CD}$, then find the coordinate of the point A when points, B, C, D are (1 , 2), (–2 , 5), (4 , 11) respectively.

(xi) Find the volume of parallelpiped if $\underline{u} = 3\underline{i} + 2\underline{k}$, $\underline{v} = \underline{i} + 2\underline{j} + \underline{k}$, $\underline{w} = -\underline{j} + 4\underline{k}$

(xii) Find a vector of length 5, in the direction opposite that of $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$

4. **Attempt any NINE parts:** 18

(i) Find the midpoint of the line segment joining the points A(3 , 1) ; B(–2 , –4). Also find the distance between them.

(ii) Find slope and inclination of the line joining the points (3 , –2) ; (2 , 7).

(iii) Find an equation of horizontal line through (7 , –9).

(iv) Find an equation of the line bisecting second and fourth quadrant.

(v) Check whether the lines $3x + 4y - 7 = 0$, $2x - 5y + 8 = 0$, $x + y - 3 = 0$ are concurrent or not ?

(Continued P/2)

(vi) Find equation of lines represented by $10x^2 - 23xy - 5y^2 = 0$

(vii) Find the measure of the angle between the lines represented by $3x^2 + 7xy + 2y^2 = 0$

(viii) Find the center and radius of the circle $x^2 + y^2 + 12x - 10y = 0$

(ix) Show that the line $3x - 2y = 0$ is tangent to the circle $x^2 + y^2 + 6x - 4y = 0$

(x) Check the position of the point (5 , 6) with respect to the circle $2x^2 + 2y^2 + 12x - 8y + 1 = 0$

(xi) Find focus and directrix of the parabola $y^2 = 8x$

(xii) Find an equation of ellipse with given data. Vertices (–1 , 1), (5 , 1) ; foci (4 , 1) and (0 , 1)

(xiii) Find equation of hyperbola with given data. foci $(0, \pm 6)$, $e = 2$

SECTION – II Attempt any THREE questions. Each question carries 10 marks.

5. (a) Evaluate: $\lim_{x \to 0} \frac{\sec x - \cos x}{x}$ 05

(b) If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, then show that $2x\frac{dy}{dx} + y = 2 \cdot \sqrt{x}$ 05

6. (a) Evaluate: $\int e^{2x} \cos 3x \, dx$ 05

(b) Find equation of line through intersection of lines $x - y - 4 = 0$, $7x + y + 20 = 0$ and parallel to line $6x + y - 14 = 0$ 05

7. (a) Evaluate: $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{\sin^{-1} x}{\sqrt{1 - x^2}} dx$ 05

(b) Maximize $f(x , y) = 2x + 5y$ subject to the constraints: $2y - x \leq 8$; $x - y \leq 4$; $x \geq 0$; $y \geq 0$ 05

8. (a) Show that $y = x^x$ has a minimum value at $x = \frac{1}{e}$. 05

(b) Find an equation of the circle which passes through the points A(5 , 10), B(6 , 9) and C(– 2 , 3) 05

9. (a) Find an equation of the ellipse with given data center (2 , 2), major axis parallel to y-axis and of length 8 units, minor axis parallel to x-axis and of length 6 units. 05

(b) Prove that by vector method. $\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \sin\beta$ 05