Intermediate Part Second

Roll No. ________

MATHEMATICS (Subjective) Group – II

Time: 02:30 Hours Marks: 80

SECTION – I

Attempt any EIGHT parts: 16

(i) Define implicit function.

(ii) Prove the identity $\text{sech}^2x = 1 - \tanh^2x$

(iii) Find $\lim\limits_{n \to 0} \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$, $x > 0$

(iv) If $y = x^4 + 2x^2 + 2$, prove that $\dfrac{dy}{dx} = 4x\sqrt{y-1}$

(v) Differentiate w.r.t. x if $y = \dfrac{2x-3}{2x+1}$

(vi) Differentiate $x^2 - \dfrac{1}{x^2}$ w.r.t. x^4

(vii) Prove that $\dfrac{d}{dx}(\sin^{-1}x) = \dfrac{1}{\sqrt{1-x^2}}$

(viii) Find $\dfrac{dy}{dx}$ if $y = x\cos y$

(ix) Differentiate $y = a^{\sqrt{x}}$

(x) Find $\dfrac{dy}{dx}$ if $y = \ell n(\tanh x)$

(xi) Define point of inflexion of a function.

(xii) Determine $f(x) = \sin x$ is increasing or decreasing in the interval $\left(0, \dfrac{\pi}{2}\right)$.

Attempt any EIGHT parts: 16

(i) Find δy and dy in $y = \sqrt{x}$, when x changes from 4 to 4.41

(ii) Evaluate $\int \sin^2 x\, dx$

(iii) Integrate by substitution $\int \dfrac{x}{\sqrt{4+x^2}}\, dx$

(iv) Find the integral $\int \dfrac{\sqrt{2}}{\sin x + \cos x}\, dx$

(v) Evaluate the integral by parts $\int \ell nx\, dx$

(vi) Find indefinite integral $\int \dfrac{1}{\sqrt{a^2 - x^2}}\, dx$ by substitution

(vii) Evaluate $\int \dfrac{2a}{x^2 - a^2}\, dx$, $x > a$ by partial fraction

(viii) What is the definition of definite integral?

(ix) Calculate the integral $\int_{-1}^{5} |x-3|\, dx$

(x) Define order of a differential equation.

(xi) What do you know about half planes?

(xii) Graph the linear inequality $2x + 3 \geq 0$

(Continued P/2)

Attempt any NINE parts: 18

(i) Find the point P on the join of A (1 , 4) and B (5 , 6) that is twice as far from A as B is from A and lies on the same side of A as B does.

(ii) Show that the points A (− 3 , 6) , B (3 , 2) and C (6 , 0) are collinear.

(iii) Find an equation of the line through the points A (− 5 , − 3) and B (9 , − 1)

(iv) Find separate equations of lines represented by $6x^2 - 19xy + 15y^2 = 0$

(v) Define eccentricity of the conic.

(vi) Find equation of parabola with focus (− 1 , 0) , vertex (− 1 , 2)

(vii) Find equation of hyperbola with foci (± 5 , 0) vertex (3 , 0)

(viii) Define a circle.

(ix) Find sum of vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ if A (1 , − 1) , B (2 , 0) , C (− 1 , 3) , D (− 2 , 2).

(x) Find a vector whose magnitude is 2 and is parallel to $-\underline{i} + \underline{j} + \underline{k}$

(xi) Find a scalar 'α' so that the vectors $2\underline{i} + \alpha\underline{j} + 5\underline{k}$ and $3\underline{i} + \underline{j} + \alpha\underline{k}$ are perpendicular.

(xii) Find area of triangle formed by P, Q, R if P (0 , 0 , 0) , Q (2 , 3 , 2) , R (− 1 , 1 , 4)

(xiii) Find α so that $\alpha\underline{i} + \underline{j}$, $\underline{i} + \underline{j} + 3\underline{k}$ and $2\underline{i} + \underline{j} - 2\underline{k}$ are coplanar.

SECTION – II Attempt any THREE questions. Each question carries 10 marks.

(a) Prove that $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x} = \log_e a$; $a > 0$ 05

(b) If $x = a(\theta - \sin\theta)$; $y = a(1 + \cos\theta)$ then prove that $y^2 \dfrac{d^2y}{dx^2} + a = 0$ 05

(a) Evaluate $\int \tan^3 x \sec x \, dx$ 05

(b) Find the equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x- and y-intercepts of each is 3. 05

(a) Evaluate $\int_0^{\sqrt{3}} \dfrac{x^3 + 9x + 1}{x^2 + 9} \, dx$ 05

(b) Indicate the solution region of the following system of linear inequalities by shading:
$3x + 7y \le 21$, $2x - y \ge -3$, $x \ge 0$ 05

(a) Find an equation of the line through the intersection of $16x - 10y - 33 = 0$, $12x + 14y + 29 = 0$ and the intersection of $x - y - 4 = 0$, $x - 7y + 2 = 0$ 05

(b) Write the equations of tangent and normal to the circle $x^2 + y^2 = 25$ at the point (4 , 3) 05

(a) Show that the ordinate at any point P of the parabola is mean proportional between the length of Latus rectum and abscissa of P. 05

(b) Prove that $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$ 05