MATHEMATICS (Subjective) Group – II

Time: 02:30 Hours Marks: 80

SECTION – I

2. Attempt any EIGHT parts: 16

(i) Determine whether the function is even or odd for $f(x) = x^{\frac{2}{3}} + 6$

(ii) Without finding inverse state domain and range of $f(x) = \frac{x-1}{x-4}$, $x \neq 4$

(iii) Evaluate $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta}$

(iv) Show that $x = at^2$, $y = 2at$ represent parametric equation of $y^2 = 4ax$

(v) Differentiate w.r.t. 'x' if $y = \frac{x^2+1}{x^2-3}$

(vi) Differentiate $x^2 - \frac{1}{x^2}$ w.r.t. x^4

(vii) Find $\frac{dy}{dx}$ if $y = xe^{\sin x}$

(viii) Find y_2 if $x^3 - y^3 = a^3$

(ix) Prove that $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

(x) Differentiate w.r.t. x, $\sin^{-1}\sqrt{1-x^2}$

(xi) Find $\frac{dy}{dx}$ if $y = e^{-2x}\sin 2x$

(xii) Find y_4 if $y = \ell n(x^2 - 9)$

3. Attempt any EIGHT parts: 16

(i) Using differentials find $\frac{dy}{dx}$ when $\frac{y}{x} - \ell nx = \ell nc$

(ii) Evaluate $\int \frac{3-\cos 2x}{1+\cos 2x}\,dx$

(iii) Evaluate the indefinite integral $\int \frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}}\,d\theta$

(iv) Evaluate $\int \frac{\sqrt{2}}{\sin x + \cos x}\,dx$

(v) Find the antiderivative of $x\ell nx$

(vi) Evaluate the given integral $\int \sec^4 x\,dx$

(vii) Solve the differential equation $\sec x + \tan y \frac{dy}{dx} = 0$

(viii) Find the area, above the x-axis and under the curve $y = 5 - x^2$ from $x = -1$ to $x = 2$

(ix) Show that the points A(0, 2), B($\sqrt{3}$, –1) and C(0, –2) are vertices of a right triangle.

(x) Convert the given equation into normal form: $15y - 8x + 3 = 0$

(xi) Find the interior angles (any two) of the triangle whose vertices are A (2, – 5), B(–4, –3), C(–1, 5)

(xii) Find an equation of each of the lines represented by $2x^2 + 3xy - 5y^2 = 0$

(Continued P/2)

4. **Attempt any NINE parts:** 18

(i) Graph the solution set of $2x + y \leq 6$ by shading.

(ii) Find the value of α so that the vectors $\alpha\underline{i} + \underline{j}$, $\underline{i} + \underline{j} + 3\underline{k}$ and $2\underline{i} + \underline{j} - 2\underline{k}$ are coplaner.

(iii) If 0 is the origin and $\overrightarrow{OP} = \overrightarrow{AB}$. Find the point P when A and B are $(-3, 7)$ and $(1, 0)$ respectively.

(iv) If $\underline{v} = 3\underline{i} - 2\underline{j} + 2\underline{k}$ and $\underline{w} = 5\underline{i} - \underline{j} + 3\underline{k}$, then find $|3\underline{v} + \underline{w}|$

(v) Find a unit vector perpendicular to plane containing $\underline{a}$ and $\underline{b}$ and $\underline{a} = -\underline{i} - \underline{j} - \underline{k}$, $\underline{b} = 2\underline{i} - 3\underline{j} + 4\underline{k}$

(vi) Compute cross product, $\underline{a} \times \underline{b}$, $\underline{b} \times \underline{a}$, if $\underline{a} = -4\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{b} = 2\underline{i} + \underline{j} + \underline{k}$

(vii) Find work done, if the point at which the constant force $\underline{F} = 4\underline{i} + 3\underline{j} + 5\underline{k}$ is applied to an object moves from $P_1(3, 1, -2)$ to $P_2(2, 4, 6)$

(viii) Find an equation of circle with center at $(\sqrt{2}, -3\sqrt{3})$ and radius is $2\sqrt{2}$

(ix) Find length of tangent drawn from point $P(-5, 4)$ to the circle $5x^2 + 5y^2 - 10x + 15y - 131 = 0$

(x) Find focus and vertex of the parabola $x^2 = 4(y - 1)$

(xi) Find center and eccentricity of $4y^2 + 12y - x^2 + 4x + 1 = 0$.

(xii) Define circle and just write its standard equation.

(xiii) Find equation of tangent to the circle $4x^2 + 3y^2 + 5x - 13y + 2 = 0$ at $\left(1, \frac{10}{3}\right)$.

SECTION – II Attempt any THREE questions. Each question carries 10 marks.

5. (a) If $f(x) = \begin{cases} \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$, find the value of k so that f is continuous at $x = 2$ 05

(b) Differentiate ab-initio w.r.t. 'x' ; $\sin\sqrt{x}$ 05

6. (a) Evaluate $\int \dfrac{\sqrt{2}}{\sin x + \cos x}\, dx$ 05

(b) Find the condition that lines $y = m_1x + c_1$, $y = m_2x + c_2$ and $y = m_3x + c_3$ are concurrent. 05

7. (a) Evaluate $\int_0^{\frac{\pi}{4}} \dfrac{\sin x - 1}{\cos^2 x}\, dx$ 05

(b) Maximize $f(x, y) = 2x + 5y$ subject to the constraints: $2y - x \leq 8$, $x - y \leq 4$, $x \geq 0$, $y \geq 0$ 05

8. (a) Find an equation of a circle which passes through $A(-3, 1)$ with radius 2 and center at $2x - 3y + 3 = 0$ 05

(b) If $\underline{a} + \underline{b} + \underline{c} = 0$ then prove that $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$ 05

9. (a) If $y = a\cos(\ell nx) + b\sin(\ell nx)$ prove that $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$ 05

(b) Find the center, foci and vertices of hyperbola $9x^2 - y^2 - 36x - 6y + 18 = 0$ 05