**Intermediate Part Second**

Roll No. ________

# MATHEMATICS ( Subjective ) Group – II

Time: 02:30 Hours Marks: 80

## SECTION – I

**2. Attempt any EIGHT parts:** 16

(i) Show that the parametric equations $x = a\sec\theta$, $y = b\tan\theta$ represent the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(ii) Express the limit $\lim_{x \to 0} (1+3x)^{\frac{2}{x}}$ in terms of e.

(iii) Evaluate $\lim_{x \to -2} \frac{2x^3 + 5x}{3x - 2}$

(iv) Differentiate $\frac{1}{x-a}$ by definition

(v) If $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$ then find $\frac{dy}{dx}$

(vi) If $y = (3x^2 - 2x + 7)^6$, then find $\frac{dy}{dx}$ by making a suitable substitution.

(vii) If $y = e^x(1 + \ell nx)$ then find $\frac{dy}{dx}$

(viii) If $y = x^2 e^{-x}$ then find $y_1$ , $y_2$

(ix) Define increasing and decreasing function.

(x) If $x = at^2$ , $y = 2at$ then find $\frac{dy}{dx}$

(xi) Graph the solution region of $4x - 3y \le 12$ , $x \ge \frac{-3}{2}$

(xii) Define optimal solution.

**3. Attempt any EIGHT parts:** 16

(i) Find $\delta y$ and $dy$ of $y = x^2 - 1$ when x changes from 3 to 3.02

(ii) Evaluate the indefinite integral $\int (\sqrt{x} + 1)^2\, dx$

(iii) Evaluate $\int \tan^2 x\, dx$

(iv) Evaluate $\int a^{x^2} x\, dx$ , $a > 0$ , $a \ne 1$

(v) Evaluate $\int \frac{-2x}{\sqrt{4 - x^2}}\, dx$

(vi) Evaluate $\int \frac{1}{(1+x^2)\tan^{-1}x}\, dx$

(vii) Find integral by parts $\int x \sin x\, dx$

(viii) Find a unit vector in direction of $\underline{v} = [3, -4]$

(ix) Write a unit vector whose magnitude is 2 and direction is same as of $\underline{v} = -\hat{i} + \hat{j} + \hat{k}$

(x) If $\underline{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ , $\underline{b} = 2\hat{i} - \hat{j} + 2\hat{k}$, find $|\underline{a} \times \underline{b}|$

(xi) Find a scalar $\alpha$ so that the vectors $2\hat{i} + \alpha\hat{j} + 5\hat{k}$ and $3\hat{i} + \hat{j} + \alpha\hat{k}$ are perpendicular.

(xii) A force $\underline{F} = 4\hat{i} - 3\hat{k}$ passes through the point A(2 , –2, 5). Find the moment of force $\underline{F}$ about the point B( 1, –3, 1).

**4. Attempt any NINE parts:** 18

(i) Show that points A(0, 2) , B($\sqrt{3}$, –1) and C(0, –2) are vertices of a right triangle.

(ii) Find h such that A(–1 , h) , B(3, 2) and C(7, 3) are collinear.

(iii) The coordinates of point P are (– 6, 9). The axes are translated through the point O′(–3, 2). Find the coordinates of point P referred to new axes.

(iv) Find equation of a straight line if its slope is 2 and y-intercepts is 5.

(v) Find the equation of the line through the points (–2, 1) and (6, –4).

( Continued P/2 )

(vi) Find the point of intersection of lines $x + 4y - 12 = 0$ and $x - 3y + 3 = 0$

(vii) Find the lines represented by $3x^2 + 7xy + 2y^2 = 0$

(viii) Find the center and radius of the circle $5x^2 + 5y^2 + 24x + 36y + 10 = 0$

(ix) Find the equation of normal to the circle $x^2 + y^2 = 25$ at (4, 3)

(x) Check position of a point (5, 6) with respect to the circle $x^2 + y^2 = 81$

(xi) Find the focus and vertex of a parabola $x^2 = 5y$

(xii) Find the equation of ellipse with foci (± 3 , 0) and minor axis of length 10

(xiii) Find foci and vertices of $x^2 - y^2 = 9$

## SECTION – II Attempt any THREE questions. Each question carries 10 marks.

5. (a) Evaluate: $\lim_{\theta \to 0} \dfrac{1-\cos p\theta}{1-\cos q\theta}$ 05

(b) Differentiate w.r.t. x, $\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$ 05

6. (a) Evaluate: $\int \dfrac{dx}{\sqrt{7-6x-x^2}}$ 05

(b) Find equation of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x- and y-intercepts of each is 3. 05

7. (a) Evaluate $\int_0^{\frac{\pi}{4}} \cos^4 t \, dt$ 05

(b) Maximize $f(x , y) = x + 3y$ subject to the constraints: $2x + 5y \leq 30$ ; $5x + 4y \leq 20$ ; $x \geq 0$ ; $y \geq 0$ 05

8. (a) If $x = \sin\theta$, $y = \sin m\theta$ show that $(1-x^2)y_2 - xy_1 + m^2y = 0$ 05

(b) Find an equation of the circle passing through the points A(1, 2) and B(1, –2) and touching to the line $x + 2y + 5 = 0$ 05

9. (a) Find center, foci, eccentricity and vertices of ellipse $x^2 + 16x + 4y^2 - 16y + 76 = 0$ 05

(b) Prove that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ 05

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