MATHEMATICS **(INTER PART-I) 321** **PAPER: I** **GROUP: I**

Time: 2:30 hours **SUBJECTIVE** **Marks: 80**

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

2. Write short answers to any EIGHT questions: **(2 x 8 = 16)**

i- Prove $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ by rules of addition.

ii- Factorize: $a^2 + 4b^2$

iii- Simplify $(2+\sqrt{-3})(3+\sqrt{-3})$

iv- If $U = \{1, 2, 3, \ldots\ldots 20\}$ and $A = \{1, 3, 5, \ldots\ldots, 19\}$ verify $A \cup A' = U$

v- Write inverse and contrapositive of the conditional $\sim p \rightarrow q$

vi- For $A = \{1, 2, 3, 4\}$, find the relation $\{(x, y) \mid x + y > 5\}$ in A

vii- Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

viii- Without expansion show that $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix} = 0$

ix- Find the inverse of matrix $A = \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

x- Write second property of cube roots of unity without proof.

xi- Find the remainder by using remainder theorem when first polynomial is divided by second polynomial $x^2 + 3x + 7$, $x + 1$

xii- Show that the roots of the equation $(p + q)x^2 - px - q = 0$ will be rational.

3. Write short answers to any EIGHT questions: **(2 x 8 = 16)**

i- Write $\frac{x^2+x-1}{(x+2)^3}$ in form of partial fractions without finding the constants.

ii- Write $\frac{1}{(x+1)^2(x^2-1)}$ in form of partial fractions without finding the constants.

iii- Write Ist four terms of the sequence $a_n = (-1)^n(2n - 3)$

iv- Find G.M. between –2 and 8

v- Find the sum of infinite geometric series $2, \sqrt{2}, 1, \ldots\ldots$

vi- Find the 9^{th} term of harmonic sequence $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\ldots$

vii- There are 5 green and 3 red balls in a box, one ball is taken out. Find the probability that the ball is green.

viii- Write in factorial form $(n + 2)(n + 1)n$

ix- How many signals can be given by 5 flags of different colours using 3 flags at a time.

x- Calculate $(0.97)^3$ by means of binomial theorem.

xi- Expand $(4 - 3x)^{1/2}$ upto three terms taking value of x such that (s.t) the expansion is valid.

xii- Determine the middle term in the expansion of $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}$

(Turn over)

4. Write short answers to any NINE questions: **(2 x 9 = 18)**

i- If $\sin\theta = -\frac{1}{\sqrt{2}}$ and the terminal arm is not in quadrant III, find the value of $\cos\theta$

ii- Verify that $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1:2:3:4$

iii- Prove that $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \cdot \operatorname{cosec}^2 A$, where $A \neq \frac{n\pi}{2}$, $n \in Z$

iv- If α, β, γ are the angles of a triangle, then prove that $\sin(\alpha+\beta) = \sin\gamma$

v- Prove that $\cos(\alpha+45^\circ) = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$

vi- Prove that $\frac{\sin A + \sin 2A}{1+\cos A+\cos 2A} = \tan A$

vii- Find the period of $\tan\frac{x}{7}$

viii- When the angle between the ground and the sun is 30°, flag pole casts a shadow of 40 m long. Find the height of the top of the flag.

ix- Find the measure of the greatest angle, if sides of the triangle are 16, 20, 33

x- Find the area of the triangle when $b = 25.4$, $\gamma = 36^\circ 41'$, $\alpha = 45^\circ 17'$

xi- Prove that $\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$

xii- Find the general solution of the trigonometric equation $\sec x = -2$

xiii- Solve the trigonometric equation and write the solution in the interval $[0, 2\pi]$ when $2\sin^2\theta - \sin\theta = 0$

SECTION II

5- (a) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$ **5**

(b) Show that the roots of $x^2 + (mx+c)^2 = a^2$ will be equal, if $c^2 = a^2(1+m^2)$ **5**

6- (a) Resolve $\frac{x^2+1}{x^3+1}$ into partial fractions. **5**

(b) If $y = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots\ldots$ and $0 < x < 2$, then prove that $x = \frac{2y}{1+y}$ **5**

7- (a) Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6? **5**

(b) If x is so small that its square and higher powers can be neglected then show that $\frac{\sqrt{1+2x}}{\sqrt{1-x}} \approx 1+\frac{3x}{2}$ **5**

8- (a) Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$ where 'θ' is not an odd multiple of $\frac{\pi}{2}$ **5**

(b) Prove without using tables/calculator that $\sin 19^\circ \cos 11^\circ + \sin 71^\circ \sin 11^\circ = \frac{1}{2}$ **5**

9- (a) P and Q are two points in line with a tree. If the distance between P and Q be 30 m and the angles of elevation of the top of the tree at P and Q be 12° and 15° respectively, find the height of the tree. **5**

(b) Show that $2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{12}{13}$ **5**