thematics (INTER PART-I) 319 GROUP: II PAPER: I
ıe: 2:30 hours SUBJECTIVE Marks: 80

ıte: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

Write short answers to any EIGHT questions: **(2 x 8 = 16)**

i- Separate into real and imaginary parts $\frac{i}{1+i}$

ii- Simplify $(i)^{101}$

iii- Show that $\forall z \in c$, $(\bar{z})^2 + z^2$ is a real number.

iv- For the conditional $p \rightarrow q$. Write its inverse and converse.

v- Define disjunction of two statements p and q

vi- If a, b are elements of a group G, then show that $(ab)^{-1} = b^{-1} a^{-1}$

vii- Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

viii- Find the value of λ if $A = \begin{bmatrix} 4 & \lambda \\ 7 & 3 \end{bmatrix}$ is singular.

ix- Define upper triangular matrix.

x- Reduce $x^{-2} - 10 = 3x^{-1}$ into quadratic form.

xi- Show that $(x^3 - y^3) = (x - y)(x - \omega y)(x - \omega^2 y)$, where ω is a cube root of unity.

xii- Show that roots of $(p + q)x^2 - px - q = 0$ are rational.

Write short answers to any EIGHT questions: **(2 x 8 = 16)**

i- Resolve $\frac{7x+25}{(x+3)(x+4)}$ into partial fractions.

ii- Define proper rational fraction.

iii- For the identity $\frac{2x-3}{x(2x+3)(x-1)} = \frac{A}{x} + \frac{B}{2x+3} + \frac{C}{x-1}$ calculate the value of A and C.

iv- Write the first four terms of the sequence $a_n = \frac{n}{2n+1}$

v- How many terms are there in A.P., in which $a_1 = 11$, $a_n = 68$, $d = 3$

vi- Sum the series $\frac{1}{1+\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1-\sqrt{x}} + \ldots\ldots\ldots$ to n terms.

vii- Find the 12^{th} term of the G.P $1 + i$, $2i$, $2(1 - i)$, ……..

viii- Find the sum of the following infinite geometric series $4 + 2\sqrt{2} + 2 + \sqrt{2} + 1 + \ldots\ldots$

ix- How many arrangements of the letters of the word 'MATHEMATICS', taken all together, can be made?

x- Prove the formula for $n = 1, 2$ $\quad 1 + 2 + 4 + \ldots\ldots + 2^{n-1} = 2^n - 1$

xi- Calculate $(2.02)^4$ by means of binomial theorem.

xii- Expand $(1 + x)^{\frac{-1}{3}}$ upto 4-terms, taking the values of x such that the expansion is valid.

(Turn over)

4. Write short answers to any NINE questions: **(2 x 9 = 18)**

i- What is the length of the arc intercepted on a circle of radius 14 cm by the arms of a central angle of $45°$?

ii- Evaluate: $\dfrac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}}$

iii- Prove that: $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$

iv- Prove that: $\tan\left(\dfrac{\pi}{4}-\theta\right)+\tan\left(\dfrac{3\pi}{4}+\theta\right) = 0$

v- Prove that: $\tan 2a = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

vi- Find the value of $\cos 2\alpha$ when $\sin\alpha = \dfrac{12}{13}$ where $0 < \alpha < \dfrac{\pi}{2}$

vii- Find the period of $\tan\dfrac{x}{3}$

viii- State law of cosines.

ix- Find the area of the triangle ABC, given three sides $a = 524$, $b = 276$, $c = 315$

x- Show that: $r_1 = s\tan\dfrac{\alpha}{2}$

xi- Prove that: $\sin^{-1}x = \dfrac{\pi}{2} - \cos^{-1}x$

xii- Find the solution of equation: $\sin x = \dfrac{-\sqrt{3}}{2}$

xiii- Solve the equation: $\sin^2 x + \cos x = 1$

SECTION II

5- **(a)** Prove that all 2 x 2 non-singular matrices over the real field form a non-abelian group under multiplication. **5**

(b) For what value of n, $\dfrac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the positive geometric mean between a and b? **5**

6- **(a)** Use Cramer's rule to solve the system: **5**

$$3x_1 + x_2 - x_3 = -4$$
$$x_1 + x_2 - 2x_3 = -4$$
$$-x_1 + 2x_2 - x_3 = 1$$

(b) The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 5 girls be formed? **5**

7- **(a)** Solve $4.2^{2x+1} - 9.2^x + 1 = 0$ **5**

(b) Find the term involving a^4 in the expansion of $\left(\dfrac{2}{x}-a\right)^9$ **5**

8- **(a)** Prove that: $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ **5**

(b) Reduce $\sin^4\theta$ to an expression involving function of multiple of θ raised to the first power. **5**

9- **(a)** The sides of a triangle are x^2+x+1, $2x+1$, x^2-1. Prove that the greatest angle of the triangle is $120°$. **5**

(b) Prove that: $\tan^{-1}\dfrac{3}{4} + \tan^{-1}\dfrac{3}{5} - \tan^{-1}\dfrac{8}{19} = \dfrac{\pi}{4}$ **5**