MATHEMATICS **(INTER PART-I) 321** **PAPER: I** **GROUP: II**

Time: 2:30 hours **SUBJECTIVE** **Marks: 80**

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

2. Write short answers to any EIGHT questions: **(2 x 8 = 16)**

i- Does the set $\{1, -1\}$ possess closure property with respect to addition and multiplication?

ii- Find the multiplicative inverse of $(\sqrt{2}, -\sqrt{5})$

iii- Show that $\forall\ Z \in C \quad Z^2 + Z^{-2}$ is a real number.

iv- Write the descriptive and tabular form of $\{x \mid x \in Q \wedge x^2 = 2\}$

v- Write the converse and inverse of $\sim p \rightarrow q$

vi- Solve the equation $ax = b$, where a, b are the elements of a group G.

vii- If A and B are square matrices of the same order, explain why in general $(A + B)(A - B) \neq A^2 - B^2$

viii- Without expansion show that $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$

ix- Find the inverse of the matrix $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

x- Solve the equation by factorization method $9x^2 - 12x - 5 = 0$

xi- Evaluate: $(1 + \omega - \omega^2)(1 - \omega + \omega^2)$

xii- Discuss the nature of the roots of the equation: $2x^2 + 5x - 1 = 0$

3. Write short answers to any EIGHT questions: **(2 x 8 = 16)**

i- Resolve into partial fractions, without finding the constants $\dfrac{x-1}{(x-2)(x+1)^3}$

ii- Write $\dfrac{1}{(x+1)^2(x^2-1)}$ in form of partial fractions without finding the constants.

iii- Which term of the arithmetic sequence OR arithmetic progression 5, 2, –1, ... is -85 ?

iv- Find the vulgar fraction equivalent to the recurring decimals $0.\overline{7}$

v- Find 9^{th} term of the harmonic sequence $\dfrac{1}{3}, \dfrac{1}{5}, \dfrac{1}{7}, \ldots\ldots$

vi- If A, G, H are the arithmetic, geometric and harmonic means between a & b respectively, Show that $G^2 = A.H$

vii- In how many ways can 4 keys be arranged on a circular key ring?

viii- Prove that ${}^nC_r = {}^nC_{n-r}$

ix- Find the value of n when, ${}^nC_{10} = \dfrac{12 \times 11}{2!}$

x- Expand by using the binomial theorem $(a + 2b)^5$

xi- Expand $(1+x)^{-1/3}$ up to 3 terms by using binomial expansion.

xii- If x is so small that its square and higher powers can be neglected then show that $\dfrac{1-x}{\sqrt{1+x}} \approx 1 - \dfrac{3}{2}x$

(Turn over)

4. Write short answers to any NINE questions: **(2 x 9 = 18)**

i- If $\operatorname{cosec}\theta = \frac{m^2+1}{2m}$ $0 < \theta < \frac{\pi}{2}$. Find the value of $\sec\theta$

ii- Evaluate: $\frac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}}$

iii- Verify that $(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$

iv- Without using calculator find the value of $\tan(1110^\circ)$

v- Prove that $\sin\left(\theta+\frac{\pi}{6}\right) + \cos\left(\theta+\frac{\pi}{3}\right) = \cos\theta$

vi- Prove that $\cot\alpha - \tan\alpha = 2\cot 2\alpha$

vii- Find the period of $\tan\frac{x}{7}$

viii- Prove that $R = \frac{abc}{4\Delta}$ using $R = \frac{a}{2\sin\alpha}$

ix- Find the measure of the greatest angle, if sides of the triangle are 16, 20, 33

x- Prove that $abc(\sin\alpha + \sin\beta + \sin\gamma) = 4\Delta s$

xi- Find the value of $\sec[\sin^{-1}(-\frac{1}{2})]$

xii- Find the general solution of the trigonometric equation $\sec x = -2$

xiii- Solve the trigonometric equation and write the solution in the interval $[0, 2\pi]$ when $2\sin^2\theta - \sin\theta = 0$

SECTION II

5- (a) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$ 5

(b) Prove that $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$ will have equal roots if $c^2 = a^2m^2 + b^2$ 5

Where $a \neq 0$, $b \neq 0$

6- (a) Resolve $\frac{2x+1}{(x+3)(x-1)(x+2)^2}$ into partial fractions. 5

(b) The sum of three numbers in an A.P is 24 and their product is 440. Find the numbers. 5

7- (a) Find the values of n and r when ${}^nC_r = 35$ and ${}^nP_r = 210$ 5

(b) If $y = \frac{2}{5} + \frac{1.3}{2!}\left(\frac{2}{5}\right)^2 + \frac{1.3.5}{3!}\left(\frac{2}{5}\right)^3 + \ldots\ldots$, then prove that $y^2 + 2y - 4 = 0$ 5

8- (a) Prove the identity $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$ 5

(b) Prove that $\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan 37^\circ$ 5

9- (a) Solve the triangle ABC if $b = 61$; $a = 32$ and $\alpha = 59^\circ 30'$ using first law of tangents and then law of sines 5

(b) Prove that $\tan^{-1}\left(\frac{1}{11}\right) + \tan^{-1}\left(\frac{5}{6}\right) = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right)$ 5