

SUBJECTIVE

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

2. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- i- Determine whether $f(x) = x\sqrt{x^2+5}$ is even or odd.
- ii- For the real valued function $f(x) = \frac{2x+1}{x-1}$ find $f^{-1}(x)$ and $f^{-1}(-1)$
- iii- If $f(x) = \begin{cases} x-1, & x < 3 \\ 2x+1, & 3 \leq x \end{cases}$ Find $\lim_{x \rightarrow 3^-} f(x)$ and $\lim_{x \rightarrow 3^+} f(x)$.
- iv- Find the derivative of $f(x) = c$ by first principle.
- v- Differentiate $y = \frac{a+x}{a-x}$ w.r.t, x
- vi- Find $\frac{dy}{dx}$ if $y = e^{x^2+1}$
- vii- Determine the values of x , for which $f(x) = x^2 + 2x - 3$ is extreme.
- viii- Show that $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$
- ix- If $y = \sin^{-1} \frac{x}{a}$ then $\frac{dy}{dx} = \frac{1}{\sqrt{a^2-x^2}}$
- x- Define a stationary point.
- xi- Define even function and give an example.
- xii- Find $\frac{dy}{dx}$ if $y = \tanh(x^2)$.

3. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- i- Use differentials, find $\frac{dy}{dx}$ if $x^2 + 2y^2 = 4$
- ii- Evaluate $\int \cos 3x \cdot \sin 2x \, dx$
- iii- Evaluate $\int \frac{\sin \theta}{1 + \cos^2 \theta} \, d\theta$
- iv- Integrate $\tan^{-1} x$ w. r. t 'x'
- v- Evaluate $\int e^x (\cos x + \sin x) \, dx$
- vi- Evaluate $\int_{-1}^2 (x + |x|) \, dx$
- vii- Find area between x-axis and curve $y = 4x - x^2$
- viii- Solve differential equation $xy + y(x-1) \, dx = 0$
- ix- Define order of differential equation.
- x- Evaluate $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} \, dx$
- xi- Define corner point.
- xii- Graph the feasible region of $3x - 2y \geq 6$

(Turn over)

4. Write short answers to any NINE questions:

- i- Show that points A(3, 1), B(-2, -3) and C(2, 2) are vertices of an isosceles triangle.
- ii- Define centroid of a triangle.
- iii- Find an equation of line through A(-6, 5) and having slope 7.
- iv- Convert into two intercept form $2x - 4y + 11 = 0$
- v- Find centre and radius of circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- vi- Determine whether the point P(-5, 6) lies outside, on or inside the circle $x^2 + y^2 + 4x - 6y - 12 = 0$
- vii- Write an equation of parabola with focus (-1, 0), vertex (-1, 2)
- viii- Find an equation of ellipse with centre (0, 0), focus (0, -3) and vertex (0, 4)
- ix- Define direction angles.
- x- If O is origin and $\overrightarrow{OP} = \overrightarrow{AB}$, find the point P where A and B are (-3, 7) and (1, 0) respectively.
- xi- Find a vector whose magnitude is 4 and is parallel to $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$
- xii- Find a and b so that the vectors $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $a\mathbf{i} + b\mathbf{j} - 2\mathbf{k}$ are parallel.
- xiii- Find a scalar α so that the vector $2\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$ and $3\mathbf{i} + \mathbf{j} + \alpha\mathbf{k}$ are perpendicular.

SECTION II

5- (a) Prove that $\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n = e$ 5

(b) Apply the Maclaurin series expansion to prove $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ 5

6- (a) Evaluate the integral $\int \frac{(a-b)x}{(x-a)(x-b)} dx$ 5

(b) Find an equation of the perpendicular bisector of the line segment joining the points A(3, 5) and B(9, 8) 5

7- (a) Find the integral $\int_0^{\sqrt{7}} \frac{3x}{\sqrt{x^2+9}} dx$ 5

(b) Graph the feasible region of the inequalities and find the corner points: 5

$$x + y \leq 5$$

$$-2x + y \geq 2$$

$$x \geq 0, y \geq 0$$

8- (a) Show that the lines $4x - 3y - 8 = 0$; $3x - 4y - 6 = 0$; $x - y - 2 = 0$ are concurrent and third line bisect the angle formed by first two. 5

(b) Find equation of circle which passes through the points A(5, 10), B(6, 9) and C(-2, 3) 5

9- (a) Find the equation of 'Ellipse' with vertices (-1, 1); (5, 1) and foci (4, 1) and (0, 1) 5

(b) Using vectors, find the area of triangle ABC whose vertices are A(1, -1, 1); B(2, 1, -1) and C(-1, 1, 2) 5