Mathematics **(INTER PART II)-421** **GROUP: I**

Time: 2:30 hours **PAPER: II**

SUBJECTIVE **Marks: 80**

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

2. Write short answers to any EIGHT questions: **(2 x 8 = 16)**

i- Show that the parametric equations $x = at^2,\ y = 2at$ represent the equation of parabola $y^2 = 4ax$

ii- Find $gof(x)$ if $f(x) = \frac{1}{\sqrt{x-1}},\quad x \neq 1\ ,\ g(x) = (x^2+1)^2$

iii- Evaluate $\lim\limits_{x \to 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$

iv- Evaluate $\lim\limits_{x \to 0} \frac{1-\cos x}{\sin^2 x}$

v- Find the derivative of $\sqrt{x}$ at $x = a$ from first principle.

vi- Differentiate $\frac{2x-3}{2x+1}$ w. r. t. 'x'.

vii- Find $\frac{dy}{dx}$ if $y^2 + x^2 - 4x = 5$

viii- Differentiate $\cos\sqrt{x} + \sqrt{\sin x}$ w. r. t. 'x'.

ix- Find $\frac{dy}{dx}$ if $y = \log_{10}(ax^2+bx+c)$

x- Find $\frac{dy}{dx}$ if $y = \frac{x}{\ell n\, x}$

xi- Find y_2 if $y = x^2 . e^{-x}$

xii- Determine the interval in which $f(x) = \cos x\ ;\ x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, is increasing.

3. Write short answers to any EIGHT questions: **(2 x 8 = 16)**

i- Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ for $xy + x = 4$

ii- Integrate $\frac{1}{\sqrt{x+a}+\sqrt{x}}$ w.r.t x

iii- Solve $\int \operatorname{cosec} x\ dx$

iv- Evaluate $\int \frac{ax}{\sqrt{a^2-x^4}}\ dx$

v- Solve $\int e^{ax}\left[a\sec^{-1}x + \frac{1}{x\sqrt{x^2-1}}\right] dx$

vi- Evaluate $\int_1^2 \frac{x}{x^2+2}\ dx$

vii- Solve $\frac{x^2+1}{y+1} = \frac{x}{y}\frac{dy}{dx}$

viii- Evaluate $\int_0^{\pi/6} x\cos x\, dx$

ix- Show that the points $A(0,2)$, $B(\sqrt{3},-1)$ and $C(0,2)$ are vertices of a right triangle.

x- Find the equation of line with slope: -5 and y-intercept is -7

xi- Show that the points $(-1,-3)$, $(1,5)$ and $(2,9)$ lie on the same straight line.

4. Write short answers to any NINE questions: **(2 x 9 = 18)**

i- Graph the solution set of linear inequality in xy-plane, $3x - 2y \geq 6$

ii- Find the equation of circle with centre $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$

iii- Find the focus and vertex of the parabola $x^2 - 4x - 8y + 4 = 0$

iv- Write an equation of parabola with axis $y = 0$, through $(2,1)$ and $(11,2)$

v- Find the coordinate of vertices of a hyperbola $\frac{y^2}{16} - \frac{x^2}{49} = 1$

vi- Find the foci of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$

vii- Find the sum of vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ given four points A(1, –1), B(2,0), C(–1,3) and D(–2,2)

viii- Find a unit vector in the direction of $\underline{v} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$

ix- Let $\underline{v} = 3\underline{i} - 2\underline{j} + 2\underline{k}$, $\underline{w} = 5\underline{i} - \underline{j} + 3\underline{k}$ find $\underline{v} - 3\underline{w}$

x- Find a vector whose magnitude is 4 and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$

xi- Find the direction cosines of $\overrightarrow{PQ}$ where P = (2, 1, 5), Q = (1, 3, 1)

xii- If $\underline{v}$ is a vector for which $\underline{v} \cdot \underline{i} = 0$, $\underline{v} \cdot \underline{j} = 0$, $\underline{v} \cdot \underline{k} = 0$, find $\underline{v}$

xiii- Find the area of a parallelogram whose vertices are A(1, 2, –1), B(4, 2, –3), C(6, –5, 2) , D(9, –5, 0)

SECTION II

5- (a) Evaluate limit by using algebraic techniques: $\lim_{x \to a} \frac{x^n - a^n}{x^m - a^m}$ **5**

(b) Find $\frac{dy}{dx}$ of the given parametric functions: $x = \frac{a(1-t^2)}{1+t^2}$; $y = \frac{2bt}{1+t^2}$ **5**

6- (a) Show that $\int \sqrt{a^2 - x^2}\, dx = \frac{a^2}{2} \sin^{-1}\frac{x}{a} + \frac{x}{2}\sqrt{a^2 - x^2} + c$ **5**

(b) Find the area of the region bounded by the triangle whose sides are **5**

$7x - y - 10 = 0$, $10x + y - 41 = 0$, $3x + 2y + 3 = 0$

7- (a) Solve the given differential equation: $\frac{1}{x}\frac{dy}{dx} = \frac{1}{2}(1 + y^2)$ **5**

(b) Maximize $f(x, y) = 2x + 5y$ subject to the constraints **5**

$2y - x \leq 8$, $x - y \leq 4$; $x \geq 0$, $y \geq 0$

8- (a) Find the centre and radius of the circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$ **5**

(b) If $\underline{a} + \underline{b} + \underline{c} = 0$, then prove that $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$ **5**

9- (a) Show that $y = \frac{\ell n\, x}{x}$ has maximum value at $x = e$ **5**

(b) Find the centre, foci and vertices of equation $9x^2 - y^2 - 36x - 6y + 18 = 0$ **5**