MATHEMATICS **Intermediate Part II Class 12th (1stA 423)** **GROUP: I**
Time: 2:30 hours **PAPER: II**
SUBJECTIVE **Marks: 80**

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

2. Write short answers to any EIGHT questions: **(2 x 8 = 16)**

i- Show that the parametric equations $x = a\cos\theta$, $y = b\sin\theta$ represent the equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

ii- Let the real valued functions 'f' and 'g' be defined by $f(x) = 2x+1$ and $g(x) = x^2-1$, obtain the expressions $fog(x)$ and $f^2(x)$

iii- Evaluate the limit $\lim\limits_{x\to 0}\frac{1-\cos 2x}{x^2}$

iv- Differentiate w.r.t.x $\frac{2x-1}{\sqrt{x^2+1}}$

v- Find $\frac{dy}{dx}$ if $x = at^2$ and $y = 2at$

vi- Find $\frac{dy}{dx}$ if $4x^2+2hxy+by^2+2gx+2fy+c=0$

vii- If $\tan y(1+\tan x) = 1-\tan x$, show that $\frac{dy}{dx} = -1$

viii- Find y_2 if $y = \ln\left(\frac{2x+3}{3x+2}\right)$

ix- Determine the intervals in which f is increasing or decreasing for the domain mentioned.
$f(x) = \sin x$; $x \in (-\pi, \pi)$

x- Find two positive integers whose sum is 30 and their product will be maximum.

xi- Define feasible region and feasible solution.

xii- Graph the feasible region of the following system of linear inequalities and find the corner points
$x+y \le 5$
$-2x+y \ge 2$
$x \ge 0$

3. Write short answers to any EIGHT questions: **(2 x 8 = 16)**

i- Find δy if $y = x^2-1$ and x changes from 3 to 3.02

ii- Evaluate $\int\frac{(1-\sqrt{x})^2}{\sqrt{x}}dx$

iii- Find the anti-derivative of $x^2\ln x$

iv- Evaluate $\int\frac{e^{m\tan^{-1}x}}{1+x^2}dx$

v- Evaluate $\int_{\pi/6}^{\pi/3}\cos t\,dt$

vi- Find the area between x-axis and the curve $y = \sin 2x$ from $x = 0$ to $x = \frac{\pi}{3}$

vii- Solve the differential equation $\frac{dy}{dx} = \frac{1+y^2}{e^{-x}}$

viii- If $\underline{v} = 3\underline{i}-2\underline{j}+2\underline{k}$ and $\underline{w} = 5\underline{i}-\underline{j}+3\underline{k}$ then find $|3\underline{v}+\underline{w}|$

(Turn over)

ix- Find direction cosines of vector $\overrightarrow{PQ}$ where P(2,1,5) and Q(1,3,1)

x- Find a vector perpendicular to each of the vectors $\underline{u} = 2\underline{i} + \underline{j} + \underline{k}$ and $\underline{v} = 4\underline{i} + 2\underline{j} - \underline{k}$

xi- Prove that $\underline{a} \times (\underline{b} + \underline{c}) + \underline{b} \times (\underline{c} + \underline{a}) + \underline{c} \times (\underline{a} + \underline{b}) = 0$

xii- Calculate the projection of $\underline{a} = \underline{i} - \underline{k}$ along $\underline{b} = \underline{j} + \underline{k}$

4. Write short answers to any NINE questions: **(2 x 9 = 18)**

i- Find the point three-fifth of the way along the line segment from A(−5,8) to B(5,3)

ii- By means of slopes show that the points (−4,6), (3,8) and (10,10) lie on the same line.

iii- Find an equation of line with x-Intercept = −9 and slope is − 4

iv- Find measure of angle between the lines represented by $10x^2 - 23xy - 5y^2 = 0$

v- Find h such that the points A(−1, h), B(3,2) and C(7,3) are collinear.

vi- Find an equation of the line through (11,−5) and parallel to a line with slope − 24.

vii- Find the co-ordinates of the point that divides the join of A(−6,3) and B(5,−2) externally in ratio 2:3

viii- Find centre and radius of the circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$

ix- Write down an equation of the parabola with focus (2,5) and directrix $y = 1$

x- Find an equation of circle of radius a and lying in 2nd Quadrant such that it is tangent to both the axes.

xi- Find focus, vertex of the parabola $x^2 = 4(y-1)$

xii- Find an equation of the hyperbola with given foci $(0, \pm 6), e = 2$

xiii- Find centre and foci of the hyperbola $\frac{y^2}{4} - x^2 = 1$

SECTION II

Note: Attempt any three (3) questions.

5- **(a)** Express the limit in terms of e $\lim\limits_{x \to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$, $x > 0$ **5**

(b) Find $\frac{dy}{dx}$ of the parametric equations $x = \frac{a(1-t^2)}{1+t^2}$, $y = \frac{2bt}{1+t^2}$ **5**

6- **(a)** Show that $\int \sqrt{a^2 - x^2}\, dx = \frac{a^2}{2} \text{Sin}^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + c$ **5**

(b) Find an equation of the line through the point (2,−9) and intersection of the lines $2x + 5y - 8 = 0$ and $3x - 4y - 6 = 0$ **5**

7- **(a)** Evaluate $\int_0^{\pi/4} \cos^4 t\, dt$ **5**

(b) Maximize $f(x,y) = 2x + 5y$ subject to the constraints $2y - x \leq 8$; $x - y \leq 4$; $x \geq 0$; $y \geq 0$ **5**

8- **(a)** If $y = (\text{Cos}^{-1}x)^2$, prove that $(1-x^2)y_2 - xy_1 - 2 = 0$ **5**

(b) Write down an equation of the circle that passes through the given points A(−7,7), B(5,−1), C(10,0) **5**

9- **(a)** Find centre, foci, eccentricity, vertices and directrices of $x^2 + 16x + 4y^2 - 16y + 76 = 0$ **5**

(b) Prove that in any ΔABC ; $a = b\text{Cos}C + c\text{Cos}B$ **5**