Time: 2:30 hours

SUBJECTIVE

GROUP: I PAPER: II Marks: 80

Note: Section I is compulsory. Attempt any three (3) questions from Section II. SECTION I

2. Write short answers to any EIGHT questions:

 $(2 \times 8 = 16)$

i- Let
$$f(x) = x^2 - x$$
, find the value of $f(x - 1)$.

ii- State the domain and range of
$$f^{-1}$$
 if $f(x) = \frac{1}{x+3}$

iii- Evaluate
$$\lim_{x \to \pi} \frac{\sin x}{\pi - x}$$

iv- Express
$$\lim_{n\to\infty} \left(1+\frac{3}{n}\right)^{2n}$$
 in term of e.

v- Differentiate
$$\frac{x^2+1}{x^2-3}$$
 w.r.t. 'x'

vi- Find
$$\frac{dy}{dx}$$
 if $x = at^2$ and $y = 2at$

vii- Prove that
$$\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$$

viii- Differentiate
$$\left(\cos\sqrt{x} + \sqrt{\sin x}\right)$$
 w.r.t 'x'

ix- Find
$$\frac{dy}{dx}$$
 if $y = \sin h^{-1}(ax + b)$

x- Find
$$\frac{dy}{dx}$$
 if $y = log_{10}(ax^2 + bx + c)$

xi- Find f'(x) if
$$f(x) = \frac{e^x}{e^{-x} + 1}$$

xii- Define a stationary point.

3. Write short answers to any EIGHT questions:

 $(2 \times 8 = 16)$

i- Use differential to find
$$\frac{dy}{dx}$$
, if $xy - \ln x = c$

ii- Evaluate
$$\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$$
, $(x > 0)$

v- Evaluate
$$\int e^x (\cos x - \sin x) dx$$

vi- Calculate
$$\int_{1}^{2} \frac{x}{x^2 + 2} dx$$

vii- Solve the differential equation
$$\frac{dy}{dx} = \frac{1-x}{y}$$

x- Convert
$$15y - 8x + 3 = 0$$
 in slope intercept form.

xii- Show that the points
$$A(-1, 2)$$
, $B(7, 5)$ and $C(2, -6)$ are vertices of right triangle.

4. Write short answers to any NINE questions:

 $(2 \times 9 = 18)$

- i- What is feasible region?
- ii- Derive equation of circle in standard form.
- iii- Write an equation of circle with centre (-3, 5) and radius 7.
- iv- Check the position of point (5, 6) with respect to circle: $2x^2 + 2y^2 + 12x 8y + 1 = 0$
- v- Find equation of hyperbola with foci $(0, \pm 9)$, directrices $y = \pm 4$.
- vi- Find the focus and directrix of the parabola if $x^2 = 5y$.
- vii- Find an equation of ellipse with foci (±3,0) and minor axis length 10.
- viii- Indicate the solution set of system of linear inequality by shading $4x 3y \le 12$; $x \ge -\frac{3}{2}$
- ix- Define equal vector, give an example.
- x- Find the magnitude and direction cosines of $\underline{v} = 4\underline{i} 5\underline{j}$
- xi- Find scalar " α " so that the vectors $2\underline{i} + \alpha \underline{j} + 5\underline{k}$ and $3\underline{i} + \underline{j} + \alpha \underline{k}$ are perpendicular.
- xii- Which vectors, if any, are parallel or perpendicular $\underline{\mathbf{u}} = \underline{\mathbf{i}} + 2\underline{\mathbf{j}} \underline{\mathbf{k}}$, $\underline{\mathbf{v}} = -\underline{\mathbf{i}} + \underline{\mathbf{j}} + \underline{\mathbf{k}}$, $\underline{\mathbf{w}} = \frac{-\pi}{2}\underline{\mathbf{i}} \pi\underline{\mathbf{j}} + \frac{\pi}{2}\underline{\mathbf{k}}$
- xiii- Prove that the vectors $\underline{i} 2\underline{j} + 3\underline{k}$, $-2\underline{i} + 3\underline{j} 4\underline{k}$ and $\underline{i} 3\underline{j} + 5\underline{k}$ are coplanar.

SECTION II

5- (a) Evaluate
$$\lim_{\theta \to 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$$

(b) If
$$\tan y(1 + \tan x) = 1 - \tan x$$
, show that $\frac{dy}{dx} = -1$

6-, (a) If
$$x = \sin \theta$$
, $y = \sin m\theta$, show that $(1-x^2)y_2 - xy_1 + m^2y = 0$

(b) Evaluate
$$\int \frac{\sqrt{2}}{\sin x + \cos x} dx$$

7- (a) Evaluate
$$\int_{0}^{\frac{\pi}{4}} \frac{1}{1+\sin x} dx$$

(b) Maximize
$$f(x,y) = 2x + 5y$$
, subject to the constraints $2y - x \le 8$; $x - y \le 4$; $x \ge 0$; $y \ge 0$.

8- (a) Find the length of the chord cut off from the line
$$2x + 3y = 13$$
 by the circle $x^2 + y^2 = 26$. 5

(b) Prove that in any
$$\triangle ABC$$
, $b^2 = c^2 + a^2 - 2ca \cos B$

9- (a) Find the interior angles of a triangle with vertices
$$A(-2,11)$$
, $B(-6,-3)$ and $C(4,-9)$ 5

(b) Find the centre, foci, eccentricity, vertices and directrices of the Ellipse $x^2 + 4y^2 = 16$

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