

SUBJECTIVE

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

2. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- Prove that $\operatorname{sech}^2 x = 1 - \tanh^2 x$
- Evaluate $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$
- Find $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi-x}$
- If $y = x^4 + 2x^2 + 2$, prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$
- Differentiate $\sin x$ w.r.t $\cot x$
- If $y = \cot^{-1}\left(\frac{x}{a}\right)$, find $\frac{dy}{dx}$
- If $f(x) = \ln(e^x + e^{-x})$, find $f'(x)$
- If $y = \tanh^{-1}(\sin x)$, find $\frac{dy}{dx}$
- If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, find y_2
- Find the interval in which $f(x)$ is increasing, $f(x) = 4 - x^2$, $x \in (-2, 2)$
- Define problem constraints.
- Graph the solution set of linear inequality in xy -plane, $3x + 7y \geq 21$

3. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- Using differentials find $\frac{dy}{dx}$, if $xy + x = 4$
- Evaluate $\int (2x+3)^{1/2} dx$
- Evaluate $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$
- Evaluate $\int e^{-x}(\cos x - \sin x) dx$
- Evaluate $\int_{-1}^2 (x + |x|) dx$
- Find the area bounded by $\cos$ function from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
- Solve $\frac{dy}{dx} = \frac{y}{x^2}$
- If O is the origin and $\overrightarrow{OP} = \overrightarrow{AB}$, find the point P when A and B are $(-3, 7)$ and $(1, 0)$ respectively.
- Find a unit vector in the direction of $\underline{v} = \underline{i} + 2\underline{j} - \underline{k}$
- Find α so that $\underline{u}$ and $\underline{v}$ are perpendicular $\underline{u} = 2\alpha\underline{i} + \underline{j} - \underline{k}$ and $\underline{v} = \underline{i} + \alpha\underline{j} + 4\underline{k}$
- Find a unit vector perpendicular to the plane containing $\underline{a}$ and $\underline{b}$, where $\underline{a} = 2\underline{i} - 6\underline{j} - 3\underline{k}$, $\underline{b} = 4\underline{i} + 3\underline{j} - \underline{k}$
- Given a force $\vec{F} = 2\underline{i} + \underline{j} - 3\underline{k}$ acting at a point $A(1, -2, 1)$. Find the moment of $\vec{F}$ about the point $B(2, 0, -2)$

(Turn over)

4. Write short answers to any NINE questions:

(2 x 9 = 18)

- i- Show that the points A(0,2), B($\sqrt{3}$, -1) and C(0,-2) are vertices of a right triangle.
- ii- The two points P(3,2) and O'(1,3) are given in XY-coordinate system. Find the XY-coordinates of P referred to the translated axes O'X and O'Y
- iii- Find K so that the line joining A(7,3), B(K,-6) and the line joining C(-4,5), D(-6,4) are parallel.
- iv- Find an equation of the vertical line through (-5,3)
- v- Find the distance from the point P(6,-1) to the line $6x - 4y + 9 = 0$
- vi- Find point of intersection of the lines $x - 2y + 1 = 0$ and $2x - y + 2 = 0$
- vii- Find measure of the angle between the lines represented by $x^2 - xy - 6y^2 = 0$
- viii- Find an equation of the circle with centre at (5,-2) and radius 4
- ix- Check the position of the point (5,6) with respect to the circle $x^2 + y^2 = 81$
- x- Find the focus and vertex of parabola $x^2 = -16y$
- xi- Find equation of ellipse with foci ($\pm 3, 0$) and minor axis of length 10
- xii- Find the centre and foci of $x^2 - y^2 = 9$
- xiii- Find the point of intersection of the given conics $x^2 + y^2 = 8$ and $x^2 - y^2 = 1$

SECTION II

Note: Attempt any three (3) questions.

- 5- (a) If $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & , \quad x \neq 2 \\ k & , \quad x = 2 \end{cases}$ 5
Find value of k so that f is continuous at $x = 2$
- (b) Differentiate $\cos x^2$ from the first principle. 5
- 6- (a) Evaluate $\int e^{2x} \cos 3x \, dx$ 5
- (b) Find the area of the region bounded by the triangle with vertices (a, b+c), (a, b-c) and (-a, c) 5
- 7- (a) Solve the differential equation $y - x \frac{dy}{dx} = 2 \left(y^2 + \frac{dy}{dx} \right)$ 5
- (b) Minimize $z = 2x + y$ subject to constraints $x + y \geq 3$, $7x + 5y \leq 35$, $x \geq 0$, $y \geq 0$ 5
- 8- (a) If $x = a(\theta + \sin \theta)$, $y = a(1 + \cos \theta)$ then, show that $y^2 = \frac{d^2y}{dx^2} + a = 0$ 5
- (b) Find an equation of the circle which passes through the points A(5,10), B(6,9) and C(-2,3) 5
- 9- (a) Find an equation of the ellipse with centre (0,0), major axis horizontal, the points (3,1), (4,0) lie on the graph. 5
- (b) Find the volume of the tetrahedron whose vertices are A(2,1,8), B(3,2,9), C(2,1,4) and D(3,3,10) 5