Roll No ____________________________ (To be filled in by the candidate)

(Academic Sessions 2015 – 2017 to 2018 – 2020)

MATHEMATICS 219-(INTER PART – I) Time Allowed : 2.30 hours

PAPER – I (Essay Type) GROUP – I Maximum Marks : 80

SECTION – I

2. Write short answers to any EIGHT (8) questions : **16**

(i) If z_1 and z_2 are complex numbers then show that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

(ii) Find out real and imaginary parts of $\left(\sqrt{3}+i\right)^3$

(iii) Factorize $a^2 + 4b^2$

(iv) Define power set of a set and give an example.

(v) Define a bijective function.

(vi) Construct truth table and show that the statement $\sim(p \to q) \to p$ is a tautology or not.

(vii) Find the matrix X if $X\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

(viii) For the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$ find cofactor A_{12}

(ix) Without expansion show that $\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix} = 0$

(x) When $x^4 + 2x^3 + kx^2 + 3$ is divided by $(x-2)$, the remainder is 1. Find the value of k.

(xi) If α, β are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then find the value of $\alpha^2 + \beta^2$

(xii) The sum of a positive number and its square is 380. Find the number.

3. Write short answers to any EIGHT (8) questions : **16**

(i) Define partial fraction.

(ii) In the identity $7x + 25 = A(x+4) + B(x+3)$, calculate values of A and B.

(iii) Resolve $\dfrac{1}{x^2-1}$ into partial fractions.

(iv) Write the first four terms of the sequence, if $a_n - a_{n-1} = n + 2$, $a_1 = 2$

(v) Which term of the arithmetic sequence 5, 2, – 1, ---- is – 85.

(vi) Find three A.Ms between 3 and 11.

(vii) If $\dfrac{1}{a}, \dfrac{1}{b}$ and $\dfrac{1}{c}$ are in G.P, show that common ratio is $\pm\sqrt{\dfrac{a}{c}}$

(viii) Insert two G.Ms between 2 and 16.

(ix) Find the value of n when ${}^nC_{10} = \dfrac{12 \times 11}{2!}$

(x) Show that $\dfrac{n^3 + 2n}{3}$ represents an integer for n = 2, 3.

(xi) Expand $\left(1 - \dfrac{3}{2}x\right)^{-2}$ upto 4 terms.

(xii) If x is so small that its square and higher power can be neglected, then show that $\dfrac{\sqrt{1+2x}}{\sqrt{1-x}} \approx 1 + \dfrac{3}{2}x$

(Turn Over)

4. Write short answers to any NINE (9) questions : 18

(i) Find ℓ, if $\theta = 65^{o}20'$, r = 18 mm

(ii) Prove $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1:2:3:4$

(iii) Prove $\cos^2\theta - \sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

(iv) Prove that $\tan 56° = \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$

(v) Prove $\frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$

(vi) Prove $\cos 20° + \cos 100° + \cos 140° = 0$

(vii) Find the period of $\tan\frac{x}{7}$

(viii) In ΔABC, $\beta = 60°$, $\gamma = 15°$, $b = \sqrt{6}$, find c.

(ix) If $a = 200$, b = 120 , $\gamma = 150°$, find the area of a triangle ABC

(x) Prove that $r_1 r_2 r_3 = rs^2$

(xi) Prove $\sin(2\cos^{-1}x) = 2x\sqrt{1-x^2}$

(xii) Solve $1 + \cos x = 0$

(xiii) Find the solutions of $\sin x = -\frac{\sqrt{3}}{2}$ in $[0, 2\pi]$

SECTION – II

Note : Attempt any THREE questions.

5. (a) Prove that all 2×2 non-singular matrices over the real field form a non-abelian group under multiplication. 5

(b) Find three, consecutive numbers in G.P whose sum is 26 and their product is 216. 5

6. (a) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 5 & -1 \\ 3 & 4 & 2 \\ 1 & 2 & -2 \end{bmatrix}$ by using row operation. 5

(b) Prove that ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$ 5

7. (a) Solve the system of equations : 5

$$12x^2 - 25xy + 12y^2 = 0$$
$$4x^2 + 7y^2 = 148$$

(b) If $y = \frac{1}{3} + \frac{1.3}{2!}(\frac{1}{3})^2 + \frac{1.3.5}{3!}(\frac{1}{3})^3 + -----$ then prove that $y^2 + 2y - 2 = 0$ 5

8. (a) Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$ where θ is not an odd multiple of $\frac{\pi}{2}$ 5

(b) If α, β, γ are the angles of a triangle ABC, then show that :

$$\cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$ 5

9. (a) The sides of a triangle are $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$. Prove that the greatest angle of the triangle is 120^{o}. 5

(b) Prove that $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$ 5