Roll No ______________________ (To be filled in by the candidate)

(Academic Sessions 2017 – 2019 to 2020 – 2022)

MATHEMATICS 221-(INTER PART – I) Time Allowed : 2.30 hours

PAPER – I (Essay Type) GROUP – I Maximum Marks : 80

SECTION – I

2. Write short answers to any EIGHT (8) questions : 16

(i) Prove that $\frac{a}{b}=\frac{ka}{kb}$, $k \neq 0$

(ii) Simplify $(5,-4)\div(-3,-8)$ and write the answer as a complex number.

(iii) Find the real and imaginary parts of $(\sqrt{3}+i)^3$

(iv) If B = { 1 , 2 , 3 }, then find the power set of B, i.e., P(B)

(v) Construct the truth table for the statement : $\sim(p\rightarrow q)\leftrightarrow(p\wedge\sim q)$

(vi) For the set A = { 1 , 2 , 3 , 4 }, find a relation in A which satisfy $\{(x,y)\,|\,y+x=5\}$

(vii) Find the matrix X , if 2X – 3A = B and $A=\begin{bmatrix}1 & -1 & 2\\ -2 & 4 & 5\end{bmatrix}$, $B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}$

(viii) Find A^{-1} if $A=\begin{bmatrix}5 & 3\\ 1 & 1\end{bmatrix}$

(ix) Without expansion, show that $\begin{vmatrix}\alpha & \beta+\gamma & 1\\ \beta & \gamma+\alpha & 1\\ \gamma & \alpha+\beta & 1\end{vmatrix}=0$

(x) Prove that sum of cube roots of unity is zero i.e., $1+\omega+\omega^2=0$

(xi) Find the numerical value of k, when the polynomial x^3+kx^2-7x+6 has a remainder of – 4 when divided by $x+2$.

(xii) Show that the roots of equation $x^2+(mx+c)^2=a^2$ will be equal if $c^2=a^2(1+m^2)$

3. Write short answers to any EIGHT (8) questions : 16

(i) Resolve $\frac{4x^2}{(x^2+1)^2(x-1)}$ into partial fractions without finding the constants.

(ii) Resolve $\frac{7x+25}{(x+3)(x+4)}$ into partial fractions without finding the constants.

(iii) Write the first four terms of the sequence, $a_n=(-1)^n n^2$

(iv) If $a_{n-3}=2n-5$, find nth term of the sequence.

(v) Insert two G.M's between 2 and 16.

(vi) Sum the infinite geometric series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+$ -----

(vii) Find the value of n, when ${}^{11}P_n=11.10.9$

(viii) Evaluate ${}^{12}C_3$

(ix) A die is rolled. What is the probability that the dots on the top are greater than 4?

(x) Check the truth of the statement 1 + 5 + 9 + ------ + (4n – 3) = n (2n – 1) for n = 1, 2

(xi) Calculate by means of binomial theorem $(2.02)^4$

(xii) If x is so small that its square and higher powers can be neglected, then show that $\frac{\sqrt{1+2x}}{\sqrt{1-x}}\approx 1+\frac{3}{2}x$

(Turn Over)

4. **Write short answers to any NINE (9) questions :** **18**

(i) Convert $54°45'$ into radians.

(ii) If $\cot\theta = \frac{15}{8}$ and the terminal arm of the angle is not in quadrant I, find the value of cosecθ.

(iii) Verify $2\sin 45° + \frac{1}{2}\operatorname{cosec}45° = \frac{3}{\sqrt{2}}$

(iv) Prove that $\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$

(v) Prove that $\tan(180° + \theta) = \tan\theta$

(vi) Express $2\sin 7\theta \sin 2\theta$ as sums or differences.

(vii) Find the period of $\tan\frac{x}{7}$

(viii) A vertical pole is 8 m high and the length of its shadow is 6m. What is the angle of elevation of the sun at that moment?

(ix) Find area of the triangle ABC if $a = 200$, $b = 120$, $\gamma = 150°$

(x) Prove that $r r_1 r_2 r_3 = \Delta^2$

(xi) Find the value of $\sec\left(\sin^{-1}(-\frac{1}{2})\right)$

(xii) Show that $r = (s-a)\tan(\frac{\alpha}{2})$

(xiii) Find the solution of $\operatorname{cosec}\theta = 2$ which lies in the interval $[0, 2\pi]$

SECTION – II

Note : Attempt any THREE questions.

5. (a) Solve by Cramer's rule
$$\begin{aligned} 2x_1 - x_2 + x_3 &= 8 \\ x_1 + 2x_2 + 2x_3 &= 6 \\ x_1 - 2x_2 - x_3 &= 1 \end{aligned}$$
5

(b) If α, β are roots of equation $ax^2 + bx + c = 0$, form the equation whose roots are $\alpha + \frac{1}{\alpha}$, $\beta + \frac{1}{\beta}$ 5

6. (a) Resolve $\frac{3x-11}{(x^2+1)(x+3)}$ into partial fraction. 5

(b) If $S_n = n(2n-1)$, then find the series. 5

7. (a) Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$ 5

(b) Use mathematical induction to prove $\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + --- + \binom{n+2}{3} = \binom{n+3}{4}$ for every positive integers n. 5

8. (a) Two cities A and B lies on the equator, such that their longitudes are 45° E and 25° W respectively. Find the distance between the two cities, taking the radius of the earth as 6400 kms. 5

(b) Prove that $\frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$ 5

9. (a) Solve the triangle ABC, if $a = 53$, $\beta = 88°36'$, $\gamma = 31°54'$ 5

(b) Prove that $\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$ 5