.oll No _____ (To be filled in by the candidate)

(Academic Sessions 2019 – 2021 to 2022 – 2024)

MATHEMATICS 223-1st Annual-(INTER PART – I) Time Allowed : 2.30 hours

PAPER – I (Essay Type) GROUP – I Maximum Marks : 80

SECTION – I

2. Write short answers to any EIGHT (8) questions : 16

(i) Show that $z^2 + \bar{z}^2$ is a real number where $z \in C$

(ii) Find the multiplicative inverse of $1-2i$

(iii) Write the descriptive and tabular form of $\{x \mid x \in P \wedge x < 12\}$

(iv) Define disjunction.

(v) If a, b are elements of a group G, solve $ax = b$

(vi) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

(vii) Find the cofactors A_{12} and A_{22} if $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$

(viii) Without expansion show that $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix} = 0$

(ix) Solve the equation $4^{1+x} + 4^{1-x} = 10$

(x) Show that the product of all the three cube roots of unity is unity.

(xi) If α, β are the roots of $ax^2 + bx + c = 0$, $a \neq 0$, find the value of $\alpha^2 + \beta^2$

(xii) The sum of a positive number and its reciprocal is $\frac{26}{5}$. Find the number.

3. Write short answers to any EIGHT (8) questions : 16

(i) Resolve $\frac{7x+25}{(x+3)(x+4)}$ into partial fraction.

(ii) If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.P., show that $b = \frac{2ac}{a+c}$

(iii) Sum the series $(x-a) + (x+a) + (x+3a) + \text{----}$ to n terms.

(iv) Find the 5th term of G.P 3, 6, 12, -----

(v) If 5 is harmonic mean between 2 and b, find b.

(vi) Find the sum to n terms of the series whose nth term is $3n^2 + n + 1$

(vii) Find the value of n when ${}^{n}P_4 : {}^{n-1}P_3 = 9:1$

(viii) How many necklaces can be made from 6 beads of different colours?

(ix) Find the value of n, when ${}^{n}C_{10} = \frac{12 \times 11}{2!}$

(x) Verify the statement $1+2+4+\text{-----}+2^{n-1} = 2^n - 1$ for $n = 1, 2$

(xi) Calculate by means of binomial theorem $(0.97)^3$ upto three decimal places.

(xii) Expand $(1-x)^{1/2}$ upto three terms.

(Turn Over)

4. Write short answers to any NINE (9) questions : **18**

(i) Convert $21.256°$ to the $D°M'S''$ form.

(ii) Verify $\sin 2\theta = 2\sin\theta\cos\theta$, when $\theta = 45°$

(iii) Prove the identity $\cos\theta + \tan\theta\sin\theta = \sec\theta$

(iv) Prove that $\sin(180°+\alpha)\sin(90°-\alpha) = -\sin\alpha\cos\alpha$

(v) Prove that $\dfrac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$

(vi) Find the values of $\cos 105°$

(vii) Find the period of $\sin\dfrac{x}{5}$

(viii) Find θ, if $\cos\theta = 0.9316$

(ix) Write any two laws of tangents.

(x) Find the value of R, if $a = 13$, $b = 14$, $c = 15$

(xi) Find the value of $\tan\left(\cos^{-1}\dfrac{\sqrt{3}}{2}\right)$

(xii) Define trigonometric equation. Give one example.

(xiii) Find the values of θ, satisfying the equation $2\sin^2\theta - \sin\theta = 0$; $\theta \in [0, 2\pi]$

SECTION – II

Note : Attempt any THREE questions.

5. (a) Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$ 5

(b) Solve the equation $x^4 - 3x^3 + 4x^2 - 3x + 1 = 0$ 5

6. (a) Resolve into partial fractions $\dfrac{5x^2 - 2x + 3}{(x+2)^3}$ 5

(b) Find the value of n and r when ${}^{n-1}C_{r-1} : {}^{n}C_{r} : {}^{n+1}C_{r+1} = 3:6:11$ 5

7. (a) If $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in G.P., show that the common ratio is $\pm\sqrt{\dfrac{a}{c}}$ 5

(b) Show that $\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \text{------} + \binom{n}{n-1} = 2^{n-1}$ 5

8. (a) Prove that $\dfrac{1}{\operatorname{cosec}\theta - \cot\theta} - \dfrac{1}{\sin\theta} = \dfrac{1}{\sin\theta} - \dfrac{1}{\operatorname{cosec}\theta + \cot\theta}$ 5

(b) Reduce $\sin^4\theta$ to an expression involving only function of multiples of θ, raised to first power. 5

9. (a) Solve the triangle using first law of tangents and then law of sines 5
$a = 36.21$, $b = 42.09$, $\gamma = 40°29'$

(b) Prove that $\sin^{-1}\dfrac{5}{13} + \sin^{-1}\dfrac{7}{25} = \cos^{-1}\dfrac{253}{325}$ 5