

SECTION – I

2. Write short answers to any EIGHT (8) questions :

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(i) Prove the rule of addition $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

(ii) Find the multiplicative inverse of $(\sqrt{2}, -\sqrt{5})$

(iii) Express the complex number $1+i\sqrt{3}$ in polar form.

(iv) Write the power set of $\{a, \{b, c\}\}$

(v) Show that the statement $p \rightarrow (p \vee q)$ is tautology.

(vi) Prove that the identity element e in a group G is unique.

(vii) If $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find a and b

(viii) If $B = \begin{bmatrix} 5 & -2 & 5 \\ 3 & -1 & 4 \\ -2 & 1 & -2 \end{bmatrix}$, find cofactor B_{21}

(ix) If A is a skew-symmetric matrix, then show that A^2 is a symmetric matrix

(x) Solve $x^{-2} - 10 = 3x^{-1}$.

(xi) If α, β are the roots of $x^2 - px - p - c = 0$ then prove that $(1+\alpha)(1+\beta) = 1 - c$

(xii) Discuss the nature of roots of the equation $x^2 - 5x + 6 = 0$

3. Write short answers to any EIGHT (8) questions :

16

(i) Define proper fraction.

(ii) If $\frac{x^2 - 10x + 13}{(x-1)(x^2 - 5x + 6)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$, find value of A

(iii) If $\frac{x}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$, find value of B

(iv) If the numbers $\frac{1}{k}, \frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in harmonic sequence, find k

(v) Find sum of infinite geometric series $2 + 1 + 0.5 + \dots$

(vi) Define geometric mean.

(vii) If 5, 8 are two A.Ms between a and b , find a and b

(viii) If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.P, show that $b = \frac{2ac}{a+c}$

(ix) Prove that ${}^nC_r = {}^nC_{n-r}$

(x) Expand $(1+x)^{\frac{-1}{3}}$ upto 3 terms.

(xi) Evaluate $\sqrt[3]{30}$ correct to three places of decimal.

(xii) Check whether the statement $5^n - 2^n$ is divisible by 3 for $n = 2, 3$ is true or false.

4. Write short answers to any NINE (9) questions :

(i) Find r , when $\ell = 56 \text{ cm}, \theta = 45^\circ$

(ii) Find the values of all trigonometric functions for -15π

(iii) Prove that $\frac{1 - \sin \theta}{\cos \theta} = \frac{\cos \theta}{1 + \sin \theta}$

(iv) Express the difference $\cos 7\theta - \cos \theta$ as product.

(v) Prove $\frac{1 - \cos \alpha}{\sin \alpha} = \tan \frac{\alpha}{2}$

(vi) Find the value of $\cos 105^\circ$ without using calculator.

(vii) Find the period of $3 \sin \frac{2x}{5}$

(viii) With usual notations prove that $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

(ix) Define in-circle of the triangle ABC.

(x) State the law of tangent. (any two)

(xi) Show that $\cos(2 \sin^{-1} x) = 1 - 2x^2$

(xii) Solve the equation for $\theta \in [0, \pi]$ $\cot^2 \theta = \frac{4}{3}$

(xiii) Solve the equation for $\theta \in [0, \pi]$ $2 \sin \theta + \cos^2 \theta - 1 = 0$

SECTION - II

Note : Attempt any THREE questions.

5. (a) If G is a group under the operation “ $*$ ” and $a, b \in G$, find the solutions of the equations : (i) $a * x = b$ (ii) $x * a = b$ 5

(b) If 7^{th} and 10^{th} terms of an H.P are $\frac{1}{3}$ and $\frac{5}{21}$ respectively, find its 14^{th} term 5

6. (a) Show that $\begin{vmatrix} a + \ell & a & a \\ a & a + \ell & a \\ a & a & a + \ell \end{vmatrix} = \ell^2 (3a + 1)$ 5

(b) Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$ 5

7. (a) If α, β are the roots of $5x^2 - x - 2 = 0$ form the equation whose roots are $\frac{3}{\alpha}$ and $\frac{3}{\beta}$ 5

(b) Use mathematical induction to prove that $n! > n^2$ for integral values of $n \geq 4$. 5

8. (a) A railway train is running on a circular track of radius 500 meters at the rate of 30 km per hour. Through what angle will it turn in 10 sec? 5

(b) Reduce $\sin^4 \theta$ to an expression involving only function of multiples of θ raised to the first power. 5

9. (a) Prove that $r_1 r_2 + r_2 r_3 + r_3 r_1 = s^2$ 5

(b) Prove that $\tan^{-1} A + \tan^{-1} B = \tan^{-1} \frac{A+B}{1-AB}$ 5