Roll No _____ (To be filled in by the candidate)

(Academic Sessions 2020 – 2022 to 2023 – 2025)

MATHEMATICS 224-1st Annual-(INTER PART – I) Time Allowed : 2.30 hours

PAPER – I (Essay Type) GROUP – II Maximum Marks : 80

SECTION – I

2. Write short answers to any EIGHT (8) questions : **16**

(i) Show that $z^2\bar{z}^2$ is a real number.

(ii) Find the modulus of $1-i\sqrt{3}$

(iii) Simplify by justifying each step $\dfrac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}$

(iv) Check the closure property w.r.t. addition and multiplication for the set $\{0, -1\}$

(v) Determine whether the statement $p \wedge \sim p$ is tautology or not.

(vi) Define semi-group.

(vii) If $A=\begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$, find $A(\bar{A})^t$

(viii) Define reduced echelon form of a matrix, with example.

(ix) If $A=\begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$, verify that $(A^{-1})^t=(A^t)^{-1}$

(x) Discuss nature of roots of $9x^2-12x+4=0$

(xi) Solve the equations $x^2+y^2=25$, $2x^2+3y^2=6$

(xii) Find the condition that one root of $x^2+px+q=0$ is square of other.

3. Write short answers to any EIGHT (8) questions : **16**

(i) Define proper rational fraction.

(ii) For the identity $\dfrac{1}{(x-1)(2x-1)(3x-1)}=\dfrac{A}{x-1}+\dfrac{B}{2x-1}+\dfrac{C}{3x-1}$ calculate the value of A.

(iii) Find the next two terms of 1, 3, 7, 15, 31, ----

(iv) How many terms are there in the A.P. in which $a_1=11$, $a_n=68$, $d=3$

(v) Find three A.Ms between $\sqrt{2}$ and $3\sqrt{2}$.

(vi) Find the 12^{th} term of $1+i, 2i, -2+2i, \text{-----}$

(vii) Show that ${}^{16}C_{11}+{}^{16}C_{10}={}^{17}C_{11}$

(viii) Evaluate ${}^{12}C_3$

(ix) What is sample space and events?

(x) State principle of mathematical induction.

(xi) Calculate $(9.98)^4$ by means of binomial theorem.

(xii) Prove that $n!>2^n-1$ for $n=4,5$

4. Write short answers to any NINE (9) questions : **18**

(i) What is length of an arc intercepted on a circle of radius 14 cm by the arms of a central angle 45^o?

(ii) Convert $54°45'$ into radians.

4. (iii) If α, β, γ are angles of triangle ABC then prove that $\cos\left(\frac{\alpha+\beta}{2}\right)=\sin\frac{\gamma}{2}$

(iv) Find the value of $\cos\frac{\pi}{12}$

(v) Express $\sin(x+30°)+\sin(x-30°)$ as a product.

(vi) Define periodic function and period of trigonometric function.

(vii) Find period of $\cos\frac{x}{6}$

(viii) Draw the graph of $y=\sin x$ from 0 to π.

(ix) State law of sines.

(x) If sides of triangle are 16, 20, 23, find its greatest angle.

(xi) Show that $r_1 = s\tan\frac{\alpha}{2}$

(xii) Find value of $\cos\left(\sin^{-1}\frac{1}{\sqrt{2}}\right)$

(xiii) Show that $\tan\left(\sin^{-1}x\right)=\frac{x}{\sqrt{1-x^2}}$

SECTION – II

Note : Attempt any THREE questions.

5. (a) Solve the system of equations by Cramer's rule : 5

$$2x+2y+z=3$$
$$3x-2y-2z=1$$
$$5x+y-3z=2$$

(b) If α, β roots of $x^2-3x+5=0$ form the equation whose roots are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$ 5

6. (a) Resolve $\frac{x^4}{1-x^4}$ into partial fractions 5

(b) The sum of an infinite geo-metric series is 9 and the sum of the squares of its terms is $\frac{81}{5}$. Find the series. 5

7. (a) Find the values of n and r when ${}^{n-1}C_{r-1} : {}^{n}C_{r} : {}^{n+1}C_{r+1} = 3:6:11$ 5

(b) If x is so small that its cube and higher powers can be neglected, then show that : $\sqrt{\frac{1-x}{1+x}} \approx 1-x+\frac{x^2}{2}$ 5

8. (a) Reduce $\cos^4\theta$ to an expression involving only function of multiples of θ, raised to the first power. 5

(b) Prove that $r_3 = 4R\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\gamma}{2}$ 5

9. (a) Show that the area of a sector of a circular region of radius r is $\frac{1}{2}r^2\theta$, where θ is the circular measure of the central angle of the sector. 5

(b) Prove that $\sin^{-1}\frac{1}{\sqrt{5}}+\cot^{-1}3=\frac{\pi}{4}$ 5