

SECTION – I

2. Write short answers to any EIGHT (8) questions :

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- (i) Define explicit function.
- (ii) Determine whether the function $f(x) = x\sqrt{x^2 + 5}$ is even or odd.
- (iii) Prove that $\lim_{x \rightarrow 0} \frac{\sqrt{x+a} - \sqrt{a}}{x} = \frac{1}{2\sqrt{a}}$
- (iv) If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, find $\frac{dy}{dx}$
- (v) Find $\frac{dy}{dx}$ if $x^2 + y^2 = 4$
- (vi) Prove that $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- (vii) Differentiate $\sin^{-1} \sqrt{1-x^2}$ w.r.t. 'x'
- (viii) Differentiate $y = a^{\sqrt{x}}$
- (ix) Prove that $\frac{d}{dx}(\cosh x) = \sinh x$
- (x) Find $\frac{dy}{dx}$ if $y = (x+1)^x$
- (xi) Define decreasing function. Give an example.
- (xii) Determine $f(x) = \cos x$ is increasing or decreasing in the interval $(\frac{\pi}{2}, \pi)$

3. Write short answers to any EIGHT (8) questions :

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- (i) What is differential coefficient?
- (ii) Evaluate $\int \frac{e^{2x} + e^x}{e^x} dx$
- (iii) Integrate by substitution $\int \frac{-2x}{\sqrt{4-x^2}} dx$
- (iv) Find the integral $\int \frac{\cos x}{\sin x \ln(\sin x)} dx$
- (v) Evaluate integral by parts $\int x \cdot \sin x dx$
- (vi) Find indefinite integral $\int a^{ax} \left[a \sec^{-1} x + \frac{1}{x\sqrt{x^2-1}} \right] dx$
- (vii) Evaluate $\int \frac{5x+8}{(x+3)(2x-1)} dx$ using partial fraction.
- (viii) Define definite integral.
- (ix) Calculate the integral $\int_0^{\frac{\pi}{4}} \sec x (\sec x + \tan x) dx$
- (x) If $\int_{-2}^1 f(x) dx = 5$, $\int_{-2}^1 g(x) dx = 4$, then evaluate $\int_{-2}^1 [3f(x) - 2g(x)] dx$

(Turn Over)

3. (xi) If a non-vertical line divides a plane into two, then write the name that two planes?

(xii) Graph the inequality $x + 3y > 6$

4. Write short answers to any NINE (9) questions :

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- Find coordinates of the point that divide the join of A (-6, 3) and B (5, -2) in the ratio 2 : 3 internally.
- Show that the triangle with vertices A (1, 1), B (4, 5) and C (12, -5) is right triangle.
- Find an equation of the line through (-4, -6) and perpendicular to the line having slope $\frac{-3}{2}$.
- Define trapezium.
- Define parabola.
- Check the position of the point (5, 6) with respect to the circle $2x^2 + 2y^2 + 12x - 8y + 1 = 0$
- Find eccentricity of the ellipse $x^2 + 4y^2 = 16$
- Find an equation of hyperbola if its foci (0, ± 9) and directrices $y = \pm 4$
- If $\vec{AB} = \vec{CD}$, find coordinates of point A. If B, C, D are (1, 2), (-2, 5), (4, 11)
- Write direction cosine of $\vec{PQ}$, if P (2, 1, 5) Q (1, 3, 1).
- Show that vectors $3\hat{i} - 2\hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} + 5\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$ form a right triangle.
- Find unit vector perpendicular to the plane of $\underline{a}$ and $\underline{b}$ if $\underline{a} = -\hat{i} - \hat{j} - \hat{k}$, $\underline{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$.
- A force $\underline{F} = 7\hat{i} + 4\hat{j} - 3\hat{k}$ is applied at P (1, -2, 3). Find its moment about the point Q (2, 1, 1).

SECTION - II

Note : Attempt any THREE questions.

5. (a) Find the values of 'm' and 'n' so that $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$

is continuous at $x = 3$

- (b) If $y = e^x \cdot \sin x$, then prove that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

6. (a) Evaluate $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$

- (b) Find an equation of the perpendicular bisector of the segment joining the points A (3, 5) and B (9, 8)

7. (a) Solve the differential equation $(x^2 - yx^2) \frac{dy}{dx} + y^2 + xy^2 = 0$

- (b) Graph the solution region of the following system of linear inequalities and find the corner points : $x + y \leq 5$, $-2x + y \leq 2$, $y \geq 0$

8. (a) Find the lines represented by each of the following and also find measure of the angle between them $x^2 + 2xy \sec \alpha + y^2 = 0$

- (b) Find the coordinates of the points of intersection of the line $2x + y + 5 = 0$ and the circle $x^2 + y^2 + 2x - 9 = 0$. Also find the length of intercepted chord.

9. (a) Find equation of parabola with elements directrix : $x = -2$, focus (2, 2)

- (b) Prove that $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ by method of vectors.