

SECTION – I

2. Write short answers to any EIGHT (8) questions :

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(i) Find the domain and range of the function g defined by : $g(x) = \sqrt{x^2 - 4}$

(ii) The real valued functions f and g are given. Find $f \circ g(x)$, if
 $f(x) = 3x^4 - 2x^2$ and $g(x) = \frac{2}{\sqrt{x}}$, $x \neq 0$

(iii) Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$

(iv) Evaluate $\lim_{x \rightarrow 1} \frac{x^3 - 3x^2 + 2x - 1}{x^3 - x}$

(v) Find $\frac{dy}{dx}$ if $x^2 - 4xy - 5y = 0$

(vi) Differentiate w.r.t. 'x' $\cot^{-1}(\frac{x}{a})$

(vii) Find $f'(x)$ if $f(x) = \sqrt{\ln(e^{2x} + e^{-2x})}$

(viii) Find y_2 if $x^3 - y^3 = a^3$

(ix) Prove that $\frac{d}{dx} (\cosec^{-1} x) = \frac{-1}{|x| \sqrt{x^2 - 1}}$

(x) Differentiate $\frac{2x - 1}{\sqrt{x^2 + 1}}$

(xi) Find the interval in which function is increasing or decreasing :
 $f(x) = \cos x \quad x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(xii) Find y_4 if $y = \sin 3x$

3. Write short answers to any EIGHT (8) questions :

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(i) Use differentials to approximate the value of $\sqrt[4]{17}$

(ii) Solve $\int \frac{dx}{\sqrt{x+1} - \sqrt{x}}$

(iii) Evaluate $\int \frac{\cot \sqrt{x}}{\sqrt{x}} dx$

(iv) Solve $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$

(v) Solve $\int e^{2x} [-\sin x + 2 \cos x] dx$

(vi) Evaluate $\int_0^{\frac{\pi}{4}} \sec x (\sec x + \tan x) dx$

(vii) Solve the differential equation $\frac{1}{x} \frac{dy}{dx} = \frac{1}{2}(1 + y^2)$

(viii) Evaluate $\int x \ln x dx$

(ix) The points $A(-5, -2)$, $B(5, -4)$ are ends of a diameter of a circle. Find centre and radius of it.

(Turn Over)

3. (x) Transform the equation $5x - 12y + 39 = 0$ into normal form.
 (xi) Find k so that the lines joining $A(7, 3)$, $B(k, -6)$ and $C(-4, 5)$, $D(-6, 4)$ are parallel.
 (xii) Find the lines represented by $2x^2 + 3xy - 5y^2 = 0$

4. Write short answers to any NINE (9) questions :

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(i) Graph the inequality $5x - 4y \leq 20$
 (ii) Find the equation of the circle with ends of diameter at $(-3, 2)$ and $(5, -6)$
 (iii) Find the centre of the circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$
 (iv) Find the length of the tangent from the point $(-5, 10)$ to the circle $5x^2 + 5y^2 + 14x - 12y - 10 = 0$
 (v) Find the coordinates of the points of intersection of the line $x + 2y = 6$ with the circle
 $x^2 + y^2 - 2x - 2y - 39 = 0$
 (vi) Find the vertex of the parabola $x^2 = 4(y - 1)$
 (vii) Find the foci of the hyperbola $\frac{y^2}{16} - \frac{x^2}{9} = 1$
 (viii) Find a unit vector in the direction of $\underline{v} = -\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}$
 (ix) Find a vector whose magnitude is 4 and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$
 (x) If \underline{v} is a vector for which $\underline{v} \cdot \underline{i} = 0$, $\underline{v} \cdot \underline{j} = 0$ and $\underline{v} \cdot \underline{k} = 0$, find \underline{v}
 (xi) If $\underline{a} + \underline{b} + \underline{c} = 0$, then prove that $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$
 (xii) Find the volume of parallelepiped for which the vectors $\underline{u} = \underline{i} - 4\underline{j} - \underline{k}$, $\underline{v} = \underline{i} - \underline{j} - 2\underline{k}$ and
 $\underline{w} = 2\underline{i} - 3\underline{j} + \underline{k}$ are three edges.
 (xiii) Give a force $\underline{F} = 2\underline{i} + \underline{j} - 3\underline{k}$ acting at a point $A(1, -2, 1)$. Find the moment
 of \underline{F} about the point $B(2, 0, -2)$

SECTION – II

Note : Attempt any THREE questions.

5. (a) Discuss the continuity of $f(x)$ at $x = c$ $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$, $c = 1$ 5

(b) Show that $\frac{dy}{dx} = \frac{y}{x}$ if $\frac{y}{x} = \tan^{-1} \frac{x}{y}$ 5

6. (a) Evaluate $\int x \sin^{-1} x \, dx$ 5

(b) Find the interior angles of the triangle with vertices $A(6, 1)$, $B(2, 7)$, $C(-6, -7)$ 5

7. (a) Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{1 + \sin x} \, dx$ 5

(b) Minimize $z = 2x + y$ subject to constraints
 $x + y \geq 3$, $7x + 5y \leq 35$; $x \geq 0$, $y \geq 0$ 5

8. (a) Prove that in any triangle ABC $b^2 = c^2 + a^2 - 2ca \cos B$. 5

(b) Find the length of the chord cut off from the line $2x + 3y = 13$ by the circle $x^2 + y^2 = 26$ 5

9. (a) If $y = (\cos^{-1} x)^2$ then prove that $(1 - x^2)y_2 - xy_1 - 2 = 0$ 5

(b) Find the points of intersection of the given conic $\frac{x^2}{18} + \frac{y^2}{8} = 1$ and $\frac{x^2}{3} - \frac{y^2}{3} = 1$ 5