.oll No ______________________ (To be filled in by the candidate)

(Academic Sessions 2020 – 2022 to 2022 – 2024)

MATHEMATICS 224-1st Annual-(INTER PART – II) Time Allowed : 2.30 hours

PAPER – II (Essay Type) GROUP – I Maximum Marks : 80

SECTION – I

2. Write short answers to any EIGHT (8) questions : **16**

(i) Prove that $\cosh^2 x - \sinh^2 x = 1$

(ii) If $f(x) = \sqrt{x+4}$ then find $f(x-1)$

(iii) Evaluate $\lim_{x\to 3} \dfrac{x-3}{\sqrt{x}-\sqrt{3}}$

(iv) Evaluate $\lim_{x\to 0} \dfrac{1-\cos 2x}{x^2}$

(v) Differentiate $y = (x^2+5)(x^3+7)$ with respect to x.

(vi) Differentiate $\dfrac{x^2+1}{x^2-3}$ with respect to x.

(vii) Find derivative of $(x^3+1)^9$ with respect to x.

(viii) Differentiate $\cos\sqrt{x} + \sqrt{\sin x}$ with respect to the variable involved.

(ix) $\dfrac{dy}{dx} = ?$ If $y = e^{x^2+1}$

(x) Find Maclaurin Series for $\sin x$

(xi) Determine the interval in which $f(x) = 4 - x^2$, $x \in (-2, 2)$ is increasing or decreasing.

(xii) Find $f'(x)$ if $f(x) = \sqrt{\ell n(e^{2x} + e^{-2x})}$

3. Write short answers to any EIGHT (8) questions : **16**

(i) Using differential to find $\dfrac{dy}{dx}$ if $xy + x = 4$

(ii) Evaluate $\int (a-2x)^{\frac{3}{2}}\, dx$

(iii) Evaluate $\int \sec x \, dx$

(iv) Evaluate $\int x \,\ell n x \, dx$

(v) Evaluate $\int_1^2 \dfrac{x}{x^2+2}\, dx$

(vi) Find the area bounded by cos function from $x = -\dfrac{\pi}{2}$ to $x = \dfrac{\pi}{2}$

(vii) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{y}{x^2}$

(viii) Find h such that A (– 1 , h) , B (3 , 2) and C (7 , 3) are collinear.

(ix) The coordinates of a point P are (3 , 2). The axes are translated through the point $O'(1,3)$. Find the coordinates of P referred to new axes.

(x) Find k so that the line joining A (7 , 3) ; B (k , – 6) and the line joining C (– 4 , 5) ; D (– 6 , 4) are parallel.

(xi) Find the point of intersection of the lines $x - 2y + 1 = 0$ and $2x - y + 2 = 0$

(xii) Find measure of the angle between the lines represented by $9x^2 + 24xy + 16y^2 = 0$

(Turn Over)

4. Write short answers to any NINE (9) questions : **18**

(i) Graph the solution set of inequality $3x-2y\geq 6$

(ii) Define feasible region.

(iii) Find the equation of circle whose ends of diameter are $(-3, 2)$ and $(5, -6)$

(iv) Find the position of the point $(5, 6)$ w.r.t the circle $2x^2+2y^2+12x-8y+1=0$

(v) Find the focus and vertex of parabola $y^2=-8(x-3)$

(vi) Find the eccentricity of ellipse $x^2+4y^2=16$

(vii) Find the centre and eccentricity of the conic $\dfrac{y^2}{4}-x^2=1$

(viii) Identify the conic represented by $4x^2-4xy+y^2-6=0$

(ix) Find the work done by a constant force $\overrightarrow{F}=2\hat{i}+4\hat{j}$, if its point of application to a body moves it from A (1 , 1) to B (4 , 6)

(x) Find the value of ' α ' such that $\alpha\underline{i}+\underline{j}$, $\underline{i}+\underline{j}+3\underline{k}$ and $2\underline{i}+\underline{j}-2\underline{k}$ are coplanar.

(xi) If $\vec{u}=2i-j+k$ and $\vec{v}=4i+2j-k$ find $\vec{u}\times\vec{v}$

(xii) Find a vector whose magnitude is 4 and is parallel to $2\underline{i}-3\underline{j}+6\underline{k}$

(xiii) If A (1 , – 1) , B (2 , 0) , C (– 1 , 3) and D (– 2 , 2) are given points, find the sum of the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$

SECTION – II

Note : Attempt any THREE questions.

5\. (a) Find m and n , so that given function f is continuous at $x=3$ **5**

$$f(x)=\begin{cases} mx & \text{if } x<3 \\ n & \text{if } x=3 \\ -2x+9 & \text{if } x>3 \end{cases}$$

(b) Prove that $y\dfrac{dy}{dx}+x=0$ if $x=\dfrac{1-t^2}{1+t^2}$, $y=\dfrac{2t}{1+t^2}$ **5**

6\. (a) If $y=e^{-ax}$, then show that $\dfrac{d^3y}{dx^3}+a^3y=0$ **5**

(b) Evaluate the indefinite integral $\int\sqrt{x^2-a^2}\,dx$ **5**

7\. (a) Solve the differential equation $2e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$ **5**

(b) Maximize $f(x,y)=x+3y$ subject to the constraints
$2x+5y\leq 30$; $5x+4y\leq 20$, $x\geq 0$, $y\geq 0$ **5**

8\. (a) Find equations of the tangents to the circle $x^2+y^2=2$ perpendicular to the line $3x+2y=6$ **5**

(b) Using vectors, prove that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ **5**

9\. (a) Find centre, foci, eccentricity, vertices and equation of directrices of $\dfrac{(y+2)^2}{9}-\dfrac{(x-2)^2}{16}=1$ **5**

(b) Find the equations of altitudes of the triangle whose vertices are
A (– 3 , 2) , B (5 , 4) , C (3 , – 8) **5**