

SECTION – I

2. Write short answers to any EIGHT (8) questions :

16

(i) Find domain and range of $f(x) = \sqrt{x+1}$

(ii) Find $f \circ f(x)$ if $f(x) = \sqrt{x+1}$

(iii) Obtain $f^{-1}(x)$ from $f(x) = 3x^3 + 7$

(iv) Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$

(v) Express $\lim_{x \rightarrow +\infty} \left(\frac{x}{1+x} \right)^x$ in terms of "e"

(vi) If $y = \frac{x^2 + 1}{x^2 - 3}$, then find $\frac{dy}{dx}$

(vii) Prove that derivative of $\tan^{-1} x$ w.r.t. "x" is $\frac{1}{1+x^2}$

(viii) Differentiate $\frac{1}{a} \sin^{-1} \left(\frac{a}{x} \right)$ w.r.t. "x"

(ix) Find $\frac{dy}{dx}$ if $y = x^2 \ln \sqrt{x}$

(x) If $y = e^{-x} (x^3 + 2x^2 + 1)$, then find $\frac{dy}{dx}$

(xi) Apply the Maclaurin's series expansion to prove that $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

(xii) Determine the interval in which $f(x) = \sin x$, $x \in (-\pi, \pi)$ is decreasing.

3. Write short answers to any EIGHT (8) questions :

16

(i) If $x^2 + 2y^2 = 16$, find $\frac{dy}{dx}$ by using differentials.

(ii) Evaluate $\int \frac{x}{x+2} dx$

(iii) Evaluate indefinite integral $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$

(iv) Evaluate $\int \ln x dx$

(v) Evaluate the definite integral $\int_{-1}^1 (x^{\frac{1}{3}} + 1) dx$

(vi) Find the area between the x-axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$

(vii) Evaluate $\int e^{-x} (\cos x - \sin x) dx$

(viii) Solve $x dy + y(x-1) dx = 0$

(ix) Show that the points A (3, 1), B (-2, -3) and C (2, 2) are vertices of an isosceles triangle

3. (x) Find an equation of line having x-intercept : -9 and slope : -4
 (xi) Show that the lines $4x - 3y - 8 = 0$, $3x - 4y - 6 = 0$ and $x - y - 2 = 0$ are concurrent.
 (xii) What is homogeneous equation?

4. Write short answers to any NINE (9) questions :

18

(i) Graph the solution set of $2x + 1 \geq 0$
 (ii) Define problem constraint.
 (iii) Find an equation of circle with centre $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$
 (iv) Find slope of tangent to $x^2 + y^2 = 5$ at $(4, 3)$
 (v) Check the position of the point $(5, 6)$ with respect to the circle $x^2 + y^2 = 81$
 (vi) Find focus and vertex of $y^2 = 8x$
 (vii) Find equation of ellipse with foci $(\pm 3, 0)$ and minor axis of length 10.
 (viii) Find equation of hyperbola with centre $(0, 0)$, focus $(6, 0)$, vertex $(4, 0)$
 (ix) Find a vector from the point A to the origin where $\vec{AB} = 4\vec{i} - 2\vec{j}$ and B $(-2, 5)$
 (x) Find α so that $|\alpha\vec{i} + (\alpha+1)\vec{j} + 2\vec{k}| = 3$
 (xi) Find the cosine of the angle θ between \underline{u} and \underline{v} ; $\underline{u} = \vec{i} - 3\vec{j} + 4\vec{k}$; $\underline{v} = 4\vec{i} - \vec{j} + 3\vec{k}$
 (xii) Prove that $\underline{a} \times (\underline{b} + \underline{c}) + \underline{b} \times (\underline{c} + \underline{a}) + \underline{c} \times (\underline{a} + \underline{b}) = 0$
 (xiii) A force $\vec{F} = 7\vec{i} + 4\vec{j} - 3\vec{k}$ is applied at P $(1, -2, 3)$. Find its moment about the point Q $(2, 1, 1)$

SECTION - II

Note : Attempt any THREE questions.

5. (a) Discuss the continuity of $f(x)$ at $x = 1$ $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 4x & \text{if } x > 1 \end{cases}$ 5
 (b) Show that $2^{x+h} = 2^x \left\{ 1 + (\ln 2)h + \frac{(\ln 2)^2 h^2}{2!} + \frac{(\ln 2)^3 h^3}{3!} + \dots \right\}$ 5

6. (a) Evaluate $\int \sqrt{4 - 5x^2} dx$ 5
 (b) Find the equation of perpendicular bisector of segment joining the points A $(3, 5)$ and B $(9, 8)$ 5

7. (a) Evaluate the integral $\int_0^{\frac{\pi}{4}} \frac{\cos \theta + \sin \theta}{2 \cos^2 \theta} d\theta$ 5
 (b) Maximize $f(x, y) = x + 3y$ subject to the constraints $2x + 5y \leq 30$; $5x + 4y \leq 20$, $x \geq 0$, $y \geq 0$ 5

8. (a) Find the interior angles whose vertices are A $(-2, 11)$, B $(-6, -3)$, C $(4, -9)$ 5
 (b) Find an equation of the circle passing through the points A $(4, 5)$, B $(-4, -3)$, C $(8, -3)$ 5

9. (a) Prove angle in a semi circle is right angle. 5
 (b) Find an equation of the tangent to the parabola $y^2 = -6x$ which is parallel to the line $2x + y + 1 = 0$. Also find point of tangency. 5