

SECTION – I**2. Write short answers to any EIGHT (8) questions :****16**

(i) Find the domain and range of $f(x) = \sqrt{x^2 - 4}$

(ii) Show that $x = a \sec \theta$, $y = b \tan \theta$ represents the equation of hyperbola.

(iii) If $f(x) = -2x + 8$, find $f^{-1}(x)$ and $f^{-1}(-1)$

(iv) Differentiate $(3-x)(x-5)$ w.r.t 'x'

(v) Find derivative of $\sqrt{\frac{1+x}{1-x}}$

(vi) If $y = x^4 + 2x^2 + 2$, prove $\frac{dy}{dx} = 4x\sqrt{y-1}$

(vii) Find the derivative of $(x^3 + 1)^9$ w.r.t. 'x'

(viii) Find $\frac{dy}{dx}$ if $y^3 - 2xy^2 + x^2y + 3x = 0$

(ix) Differentiate w.r.t. variable involved of $\tan^3 \theta \sec^2 \theta$

(x) Find $\frac{dy}{dx}$ if $y = a^x$

(xi) Define feasible region.

(xii) Graph the feasible region $2x - 3y \leq 6$ $x \geq 0$, $y \geq 0$

3. Write short answers to any EIGHT (8) questions :**16**

(i) Using differentials to find $\frac{dy}{dx}$ if $xy - \ln x = c$

(ii) Evaluate $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$

(iii) Evaluate $\int \frac{e^x}{e^x + 3} dx$

(iv) Evaluate $\int \ln x dx$

(v) Evaluate $\int_{-6}^2 \sqrt{3-x} dx$

(vi) Find the area bounded by cos function from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$

(vii) Solve the differential equation $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

(viii) Find the magnitude of the vector $\underline{u} = \hat{i} + \hat{j}$

(ix) Find direction cosines of $\vec{v} = 3\hat{i} - \hat{j} + 2\hat{k}$

(x) Calculate the projection of \vec{b} along \vec{a} if $\vec{a} = \hat{i} - \hat{k}$; $\vec{b} = \hat{j} + \hat{k}$

(xi) If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$; $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, find $\vec{b} \times \vec{a}$

(xii) Prove that the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$, $-2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar.

4. Write short answers to any NINE (9) questions :

(i) Find the equation of the straight line whose slope is 2 and y-intercept is 5.

(ii) Using slopes, show that the triangle with its vertices A (6, 1), B (2, 7) and C (- 6, - 7) is a right triangle.

(iii) Find an equation of the line through (- 4, 7) and parallel to the line $2x - 7y + 4 = 0$

(iv) Find h such that A (- 1, h), B (3, 2) and C (7, 3) are collinear.

(v) Write intercepts form of equation of straight line.

(vi) Check whether the following lines are concurrent or not
 $3x - 4y - 3 = 0$
 $5x + 12y + 1 = 0$
 $32x + 4y - 17 = 0$

(vii) Find the slope and inclination of the line joining points (- 2, 4) and (5, 11)

(viii) Find an equation of circle with centre at $(\sqrt{2}, - 3\sqrt{3})$ and radius $2\sqrt{2}$

(ix) Define focus and directrix of the parabola.

(x) Find the centre and foci of the ellipse $x^2 + 4y^2 = 16$

(xi) Find equation of tangent to $y^2 = 4ax$ at (x_1, y_1)

(xii) Show that the equation $5x^2 + 5y^2 + 24x + 36y + 10 = 0$ represents a circle. Find its centre.

(xiii) Find an equation of the ellipse with given data : Foci (0, - 1) and (0, - 5) and major axis of length 6.

SECTION - II

Note : Attempt any THREE questions.

5. (a) If θ is measured in radians then prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ 5

(b) Find $\frac{dy}{dx}$ if $y = (1 + 2\sqrt{x})^3 \cdot x^{\frac{3}{2}}$ 5

6. (a) Evaluate $\int \ln(x + \sqrt{x^2 + 1}) dx$ 5

(b) Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x-intercept and y-intercept of each is 3. 5

7. (a) Solve the differential equation $2e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$ 5

(b) Maximize $f(x, y) = x + 3y$ subject to constraints
 $2x + 5y \leq 30, 5x + 4y \leq 20, x \geq 0, y \geq 0$ 5

8. (a) If $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$ 5

(b) Find equations of tangents to the circle $x^2 + y^2 = 2$ perpendicular to the line $3x + 2y = 6$ 5

9. (a) Show that the equation $9x^2 - 18x + 4y^2 + 8y - 23 = 0$ represents an ellipse. Find its elements and sketch its graph. 5

(b) Prove that in any triangle ABC $c = a \cos B + b \cos A$ 5