Roll No _____ (To be filled in by the candidate)

(Academic Sessions 2020 – 2022 to 2022 – 2024)

MATHEMATICS 224-1st Annual-(INTER PART – II) Time Allowed : 2.30 hours

PAPER – II (Essay Type) GROUP – II Maximum Marks : 80

SECTION – I

2. Write short answers to any EIGHT (8) questions : **16**

(i) Given that $f(x)=\cos x$ find $\frac{f(a+h)-f(a)}{h}$ and simplify.

(ii) If $f(x)=(-x+9)^3$, find $f^{-1}(x)$

(iii) By rationalizing, find $\lim_{x\to 0}\frac{\sqrt{x+a}-\sqrt{a}}{x}$

(iv) Write down the domain and range of $f(x)=2x-5$

(v) Calculate derivative of $f(x)=x^{2/3}$ at $x=8$

(vi) Find derivative of $\frac{1+x}{1-x}$ *w.r.t.* x

(vii) If $y=x^4+2x^2+2$, find $\frac{dy}{dx}$

(viii) Find $\frac{dy}{dx}$ of implicit function $x^2-4xy-5y=0$

(ix) Apply chain rule to find $\frac{dy}{du}$ if $y=x^2+\frac{1}{x^2}$ and $u=x-\frac{1}{x}$

(x) Differentiate $\sin^2 x$ *w.r.t* $\cos^4 x$

(xi) Find $f'(x)$ if $f(x)=x^3 e^{\frac{1}{x}}$

(xii) Find y_2 if $y=x^2.e^{-x}$

3. Write short answers to any EIGHT (8) questions : **16**

(i) Using differential to find $\frac{dx}{dy}$ of $x^4+y^2=xy^2$

(ii) Evaluate $\int (2x+3)^{\frac{1}{2}}\,dx$

(iii) Evaluate $\int x\sqrt{x-a}\,dx$

(iv) Evaluate $\int (\ell nx)^2\,dx$

(v) Evaluate $\int_1^2 \left(x+\frac{1}{x}\right)^{\frac{1}{2}}\left(1-\frac{1}{x^2}\right)dx$

(vi) Find the area bounded by cos function from $x=-\frac{\pi}{2}$ to $x=\frac{\pi}{2}$

(vii) Solve $\frac{dy}{dx}+\frac{2xy}{2y+1}=x$

(viii) Find the mid-points of the line joining the two points A (– 8 , 3) , B (2 , – 1).

(ix) Find h such that the points A (– 1 , h) , B (3 , 2) and C (7 , 3) are collinear.

(x) In the triangle A (8 , 6) , B (– 4 , 2) , C (– 2 , – 6), find the slope of altitude of triangle.

(xi) Using slopes, show that the triangle with vertices A (6 , 1) , B (2 , 7) , C (– 6 , – 7) is a right triangle.

(xii) Find the point of intersection of the lines $x+4y-12=0$, $x-3y+3=0$

4. Write short answers to any NINE (9) questions : **18**

(i) Define feasible region.

(ii) Graph the solution set of $5x - 4y \leq 20$

(iii) Write the standard and general equation of circle.

(iv) Find centre and radius of $5x^2 + 5y^2 + 24x + 36y + 10 = 0$

(v) Check the position of the point (5 , 6) with respect to the circle $x^2 + y^2 = 81$

(vi) Find the length of the tangent drawn from the point (– 5 , 4) to the circle $5x^2 + 5y^2 - 10x + 15y - 131 = 0$

(vii) Find foci and eccentricity of ellipse $x^2 + 4y^2 = 16$

(viii) Find the points of intersection of $x^2 + y^2 = 8$ and $x^2 - y^2 = 1$

(ix) If $\underline{u} = 2\underline{i} - 7\underline{j}$, $\underline{v} = \underline{i} - 6\underline{j}$ and $\underline{w} = -\underline{i} + \underline{j}$, find $\frac{1}{2}\underline{u} + \frac{1}{2}\underline{v} + \frac{1}{2}\underline{w}$

(x) Find a vector whose magnitude is 4 and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$

(xi) Find α so that the vector $\underline{u}$ and $\underline{v}$ are perpendicular ; $\underline{u} = \alpha\underline{i} + 2\alpha\underline{j} - \underline{k}$ and $\underline{v} = \underline{i} + \alpha\underline{j} + 3\underline{k}$

(xii) Find the area of parallelogram whose vertices are A (1 , 2 , – 1) ; B (4 , 2 , – 3) ; C (6 , – 5 , 2) ; D (9 , – 5 , 0)

(xiii) Prove that $\underline{u}.(\underline{v} \times \underline{w}) + \underline{v}.(\underline{w} \times \underline{u}) + \underline{w}.(\underline{u} \times \underline{v}) = 3\underline{u}.(\underline{v} \times \underline{w})$

SECTION – II

Note : Attempt any THREE questions.

5. (a) Evaluate $\lim_{x \to 0} \frac{\sec x - \cos x}{x}$ 5

(b) Find the derivative w.r.t. x $\sin\sqrt{\frac{1+2x}{1+x}}$ 5

6. (a) If $y = (\cos^{-1} x)^2$, prove that $(1 - x^2)\, y_2 - xy_1 - 2 = 0$ 5

(b) Evaluate $\int \frac{2x}{1 - \sin x}\, dx$ 5

7. (a) Find the area between the x-axis and the curve $y = \sqrt{2ax - x^2}$ when $a > 0$ 5

(b) Maximize $f(x, y) = 2x + 5y$ subject to the constraints $2y - x \leq 8$; $x - y \leq 4$, $x \geq 0$, $y \geq 0$ 5

8. (a) Find equation of the circle passing through the points A (3 , – 1) , B (0 , 1) and having centre at $4x - 3y - 3 = 0$ 5

(b) Use vectors to prove that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ 5

9. (a) Mid-points of sides of triangle are (1 , – 1) , (– 4 , – 3) and (– 1 , 1). Find coordinates of vertices of triangle. 5

(b) Show that equation of parabola with focus at $(a\cos\alpha, a\sin\alpha)$ and directrix $x\cos\alpha + y\sin\alpha + a = 0$ is $(x\sin\alpha - y\cos\alpha)^2 = 4a(x\cos\alpha + y\sin\alpha)$ 5