MATHEMATICS (Subjective) Group – I

Time: 02:30 Hours Marks: 80

SECTION – I

2. **Attempt any EIGHT parts:** 16

(i) Show that parametric equations $x = a\cos\theta$, $y = b\sin\theta$ represent the equation of Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(ii) If $f(x)=\sqrt{x+1}$, $g(x)=\frac{1}{x^2}$, find $(fog)(x)$

(iii) Evaluate the limit: $\lim\limits_{x\to 0}\ \frac{\sin ax}{\sin bx}$

(iv) Discuss the continuity of $f(x)=\begin{cases}2x+5, & x\le 2\\ 4x+1, & x>2\end{cases}$ at $x=2$

(v) Use definition to find the derivative of $x(x-3)$ w.r.t. 'x'

(vi) Differentiate $x^4+2x^3+x^2$ w.r.t. 'x'

(vii) Differentiate $(1+x^2)^n$ w.r.t. x^2

(viii) Find $\frac{dy}{dx}$ when $x = y\sin y$

(ix) If $y = e^{-2x}\sin 2x$, find $\frac{dy}{dx}$

(x) Find $\frac{dy}{dx}$ when $y=\sinh^{-1}(x^3)$

(xi) Use Maclaurin Series to prove that $e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots$

(xii) Find the interval where $f(x)=4-x^2$, $x\in(-2, 2)$ is increasing or decreasing in the given domain.

3. **Attempt any EIGHT parts:** 16

(i) Use differentials, find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ of $x^2+2y^2=16$

(ii) Evaluate $\int\sin^2 x\,dx$

(iii) Find $\int\frac{dx}{x(\ell n2x)^2}$

(iv) Evaluate $\int\sin^{-1}x\,dx$

(v) Evaluate $\int_1^2 \ell nx\,dx$

(vi) Find area above the x-axis, bounded by curve $y^2=3-x$ from $x=-1$ to $x=2$

(vii) Solve differential equation $1+\cos x\tan y\frac{dy}{dx}=0$

(viii) Find point three-fifth of way along the line segment from A(– 5 , 8) to B(5 , 3)

(ix) Two points P and O′ are given in xy-coordinate system. Find XY-coordinates of P. $P\left(\frac{3}{2},\frac{5}{2}\right)$; $O'\left(-\frac{1}{2},\frac{7}{2}\right)$

(x) Find an equation of line through $(-4,-6)$ and perpendicular to the line having slope $-\frac{3}{2}$

(xi) Express the system $3x + 4y - 7 = 0$, $2x - 5y + 8 = 0$, $x + y - 3 = 0$ in matrix form and check whether three lines are concurrent.

(xii) Find lines represented by $x^2-2xy\sec\alpha+y^2=0$

(Continued P/2)

4. **Attempt any NINE parts:** 18

(i) Graph the solution set of linear inequality $5x - 4y \leq 20$ in xy-plane.

(ii) Define corner point of solution region.

(iii) Find center and radius of the circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$

(iv) Find equation of parabola whose focus is F(–3, 4) and directrix is $3x - 4y + 5 = 0$

(v) Find length of the tangent drawn from the point (– 5 , 4) to the circle $5x^2 + 5y^2 - 10x + 15y - 131 = 0$

(vi) Find focus and vertices of Ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

(vii) Find equation of tangent to conic $y^2 = 4ax$ at $(at^2, 2at)$

(viii) Find equation of hyperbola with center (0 , 0) , focus (6 , 0) vertex (4 , 0).

(ix) If O is origin and $\overrightarrow{OP} = \overrightarrow{AB}$, find the point P when A and B are (–3 , 7) and (1 , 0) respectively.

(x) Find direction cosines of vector $\underline{v} = \underline{i} - \underline{j} - \underline{k}$

(xi) Find cosine of the angle θ between vectors $\underline{u} = 3\underline{i} + \underline{j} - \underline{k}$, $\underline{v} = 2\underline{i} - \underline{j} + \underline{k}$

(xii) A force $\underline{F} = 7\underline{i} + 4\underline{j} - 3\underline{k}$ is applied at P(1, – 2, 3), find its moment about Q(2, 1, 1)

(xiii) Find the volume of the parallelepiped determined by $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$, $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$, $\underline{w} = \underline{i} - 7\underline{j} - 4\underline{k}$

SECTION – II Attempt any THREE questions. Each question carries 10 marks.

5. (a) If $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & , x \neq 2 \\ k & , x = 2 \end{cases}$, find the value of 'k' so that f is continuous at x = 2. 05

(b) Prove that $y\frac{dy}{dx} + x = 0$ if $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$ 05

6. (a) Show that $y = x^x$ has minimum value at $x = \frac{1}{e}$ 05

(b) Evaluate: $\int \frac{dx}{(1+x^2)^{3/2}}$ 05

7. (a) Find the area between x-axis and curve $y = \sqrt{2ax - x^2}$, when $a > 0$ 05

(b) Minimize $z = 3x + y$; subject to constraints $3x + 5y \geq 15$; $x + 3y \geq 9$, $x, y \geq 0$ 05

8. (a) Find the length of the chord cut off from the line $2x + 3y = 13$ by the circle $x^2 + y^2 = 26$ 05

(b) Use vector method to show that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ 05

9. (a) Write an equation of the parabola with given elements:
Focus (–3 , 1) ; directrix $x - 2y - 3 = 0$ 05

(b) Find the distance between the given parallel lines. Sketch the lines. Also find an equation of the parallel line lying midway between them:
$3x - 4y + 3 = 0$; $3x - 4y + 7 = 0$ 05