

MATHEMATICS (Subjective) Group - II

Time: 02:30 Hours Marks: 80

SECTION - I

2. Attempt any EIGHT parts:

(i) If $f(x) = \begin{cases} x+2, & x \leq -1 \\ c+2, & x > -1 \end{cases}$, then find c so that $\lim_{x \rightarrow -1} f(x)$ exists.

(ii) Evaluate $\lim_{x \rightarrow \infty} \left(\frac{x}{1+x} \right)^x$

(iii) If $g(x) = \frac{3}{x-1}$, $x \neq 1$; then find $gog(x)$

(iv) Determine whether $f(x) = \frac{3x}{x^2+1}$ is even or odd.

(v) Differentiate $\frac{2x-3}{2x+1}$ w.r.t x

(vi) Find $\frac{dy}{dx}$ if $x = \theta + \frac{1}{\theta}$ and $y = \theta + 1$

(vii) Differentiate $\cos \sqrt{x} + \sqrt{\sin x}$ w.r.t x

(viii) Differentiate $\sqrt{\tan x}$ w.r.t x

(ix) Find $f'(x)$ if $f(x) = \ln(e^x + e^{-x})$

(x) Find y_2 if $x^3 - y^3 = a^3$

(xi) Prove that $\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4} - \frac{x^6}{6} + \dots$

(xii) Determine the interval in which $f(x) = \sin x$ is decreasing; $x \in (-\pi, \pi)$

3. Attempt any EIGHT parts:

(i) Find dy and δy for the function $y = \sqrt{x}$ when x changes from 4 to 4.41

(ii) Evaluate $\int (3x^2 - 2x + 1) dx$

(iii) Evaluate the integral $\int \frac{1-x^2}{1+x^2} dx$

(iv) Evaluate $\int x^3 \ln x dx$

(v) Evaluate $\int \frac{2x}{x^2 - a^2} dx$

(vi) Solve the definite integral $\int_{-1}^3 (x^3 + 3x^2) dx$

(vii) Find the area between x-axis and the curve $y = \cos \frac{1}{2}x$ from $x = -\pi$ to $x = \pi$

(viii) Find 'h' such that points $A(-1, h)$, $B(3, 2)$ and $C(7, 3)$ are collinear.

(ix) Find the slope and inclination of the line joining the points $(4, 6)$ and $(4, 8)$.

(x) Find the equation of line through $(-4, 7)$ and parallel to the line $2x - 7y + 4 = 0$

(xi) Check whether the lines $4x - 3y - 8 = 0$; $3x - 4y - 6 = 0$ and $x - y - 2 = 0$ are concurrent or not.

(xii) Find the angle between the pair of lines $x^2 + 2xy \sec \alpha + y^2 = 0$

(Continued P/2)

4. Attempt any NINE parts:

- (i) Indicate solution set of linear inequalities $3x + 7y \geq 21$, $x - y \leq 2$
- (ii) Define optimal solution.
- (iii) Find center and radius of the circle $x^2 + y^2 - 6x + 4y + 13 = 0$
- (iv) Find length of tangent drawn from point $(-5, 4)$ to the circle $5x^2 + 5y^2 - 10x + 15y - 131 = 0$
- (v) Find the vertex and directrix of parabola $x^2 = 5y$
- (vi) Find equation of ellipse with data vertices $(-1, 1), (5, 1)$ Foci $(4, 1), (0, 1)$
- (vii) Find equation of hyperbola with data Foci $(0, \pm 9)$, directrices $y = \pm 4$
- (viii) Find equation of normal to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(a \sec \theta, b \tan \theta)$
- (ix) Find unit vector in the direction of vector $\underline{v} = -\frac{\sqrt{3}}{2} \underline{i} - \frac{1}{2} \underline{j}$
- (x) Find direction cosines of vector $\underline{v} = 6\underline{i} - 2\underline{j} + \underline{k}$
- (xi) Show that the set of points $P(1, 3, 2)$, $Q(4, 1, 4)$ and $R(6, 5, 5)$ forms a right triangle.
- (xii) Compute cross product $\underline{b} \times \underline{a}$ if $\underline{a} = 3\underline{i} - 2\underline{j} + \underline{k}$, $\underline{b} = \underline{i} + \underline{j}$
- (xiii) Prove that vectors $\underline{i} - 2\underline{j} + 3\underline{k}$, $-2\underline{i} + 3\underline{j} - 4\underline{k}$, $\underline{i} - 3\underline{j} + 5\underline{k}$ are coplanar.

SECTION – II

Attempt any THREE questions. Each question carries 10 marks.

5. (a) If $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$, find the value of 'k' for which f is continuous at $x = 2$. 05

(b) Find $\frac{dy}{dx}$, if $y = x \sin^{-1} \left(\frac{x}{a} \right) + \sqrt{a^2 - x^2}$ 05

6. (a) Show that $y = x^x$ has minimum value at $x = \frac{1}{e}$ 05

(b) Evaluate the indefinite integral $\int \sqrt{4 - 5x^2} dx$ 05

7. (a) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x(2 + \sin x)} dx$ 05

(b) Graph the feasible region of linear inequalities and find corner points:
 $2x + 3y \leq 18$; $2x + y \leq 10$; $x + 4y \leq 12$ 05

8. (a) Find an equation of circle passes through $A(5, 1)$ and tangent to line $2x - y - 10 = 0$ at $B(3, -4)$ 05

(b) Prove that the angle in a semi-circle is a right angle. 05

9. (a) Find the focus, vertex and directrix of the parabola; $y^2 = -8(x - 3)$ 05

(b) Find the lines represented by $9x^2 + 24xy + 16y^2 = 0$ and also find measure of the angle between them. 05

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